Self-fulfilling Prophecies in the Transition to Clean Technology

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Abstract

Technological lock-in has been a standard explanation for the lack of clean innovation, but is hard to reconcile with forward-looking investors who anticipate the eventual switch to clean technologies. We provide an alternative explanation: technical change is the outcome of coordinated actions among innovators and self-fulfilling prophecies can lead to delayed low-carbon transition. We analyse a standard directed technical change model with clean and dirty goods. We find that when the two are good substitutes, two stable steady states can co-exist, each allowing multiple transitional paths. Optimal low-carbon transition requires a coordination device in addition to a Pigouvian carbon tax.

JEL codes: C62, O33, O38, O44, Q55

Keywords: Directed technical change, strategic complementarity, multiple equilibria, delayed low-carbon transition, stranded asset

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1 Introduction

As ever more frequent natural disasters drive home the message of an urgent need for climate actions, climate risk is becoming a mainstream consideration. Anticipating more stringent climate policies and recognizing the inevitability of a transition towards cleaner production, more and more companies put forward their net-zero pledges (BEIS, 2021; Thorbecke, 2021). Nonetheless, the warnings of asset stranding (van der Ploeg and Rezai, 2020) highlight the concern that the private sector may not be transitioning to clean technologies promptly and optimally. The recent evidence of a declining share of clean innovation (see Acemoglu et al., 2019) serves as a case in point.

One explanation for the slow transition is path dependency and technological lock-in, as emphasized by the directed technical change (DTC) literature (see for example Acemoglu et al., 2012). Companies are reluctant to give up their firm position in polluting industries and forego historically established markets. However, while ignoring history is costly, ignoring the future might be even more so, particularly for long-lived assets and forward-looking decisions such as innovation and investments. Is continued investment in polluting technologies thus simply a reflection of firms' myopia?

Rather than myopia driving the lack of investment, a coordination failure might be at the root of it. As the literature on coordination failures has pointed out in a different context (see for example Krugman, 1991; Matsuyama, 1991), individual investors prefer to direct their investment to markets that are believed to be big and profitable in future, but the size of future markets depends on aggregate investments of these investors themselves. For example, in more innovative markets firms have access to better technology options and lower cost inputs. In this situation complementarities in firms' investment decisions create multiple equilibria. If all believe a particular market is the most profitable one to invest in, investors flock in this market and the resulting bigger market justifies their collective choice. But another market would have been targeted with different beliefs. Along this mechanism, self-fulfilling prophecies may strengthen brown investment and counteract green policies.

This paper develops a DTC model in which two goods, produced using clean and polluting technologies respectively, compete for market shares and revenues, while monopolistic firms supplying these technologies invest in R&D in order to increase their firm value. How much an individual firm benefits from innovation is measured by the discounted value of its profits from future sales. Profits depend on both the total consumer spending in the sector and the firm's share in it. The latter depends on the quality of the firm's product relative to average quality in the sector. Innovation in a sector shifts consumer spending towards it as long as the two goods are gross substitutes. Investment by firms within a sector thus generates a demand externality. Innovation by other firms in the same sector also lowers one's own revenue share, leading to a business stealing effect. For sufficiently good substitutability, the former outweighs the latter and innovations by firms within a sector become strategic complements: innovation by one firm raises the return to innovation for other firms in the sector, which in turn raises their own incentive to innovate.

In this macro-economic dynamic setting, expectations or beliefs about future environmental-friendliness of innovation can overturn the lock-in in polluting technology and create self-fulfilling prophecies. In particular, we find that two stable steady-state equilibria can co-exist – one in which dirty technologies dominate and one in which only clean technologies are used – and that the selection between the two involves a coordination problem. In addition to the selection of the steady state, the transition towards a steady state is itself subject to coordination. We find that multiple transitional paths exist to each of the steady states that are consistent with rational deterministic expectations, i.e. expectations that can be rationalized in an equilibrium without stochastic shocks. Consequently, even if climate policy can eliminate the undesirable steady state, it may not rule out delays and excess asset-stranding during the transition towards the clean steady state, as self-fulfilling prophecies continue to exist for the selection among the multiple transitional paths.

Our result reconciles the observation that firms continue to invest in polluting technologies despite the inevitability of more stringent climate policy and the transition towards clean technologies. In terms of climate policy, our results show that a Pigouvian carbon tax is insufficient for achieving the optimal low-carbon transition.

As the optimal transition requires not only the correction of climate externality but also the coordination for the appropriate transitional path, a separate coordination device is necessary, be it a tax for R&D in the polluting technology or a quantity restriction such as from an emission trading system. Our results thus lend support to the simultaneous use of multiple policy instruments rather than relying on a Pigouvian carbon tax alone.

Related literature Our paper contributes to a few strands of literature. Firstly, our paper is related to endogenous growth models featuring multiple equilibria. Examples relying on the increasing returns and/or externalities to generate indeterminacy include Benhabib and Perli (1994), Benhabib and Farmer (1994), Boldrin and Rustichini (1994), Howitt and McAfee (1988), and Benhabib et al. (2008), while rational-expectation-based indeterminacy can be found in Cozzi (2005), Cozzi (2007) and Gil (2013). Most recent examples with a focus on environmental issues include van der Meijden and Smulders (2017) and Bretschger and Schaefer (2017).[1] As is common among all these models, some sort of strategic complementarity is needed for the existence of multiple equilibria. In our paper, strategic complementarity arises since firms' profits in a sector benefit from a higher demand for the sector's goods which in turn depends on innovation by all firms in this sector.

Secondly, the model setup in our paper follows the tradition of the DTC literature (Acemoglu et al., 2012; Hassler et al., 2021; Lemoine, 2022). Our model departs from the tradition in this literature by assuming an infinite patent length instead of a one-period patent. With this assumption of long-lived assets, our paper emphasizes the role of beliefs about future profitability in the decision of R&D investment.[2] However, as we show in Appendix OA6, the existence of strategic complementarity does not necessitate a long patent length. Our paper thus also serves as a caution that multiple equilibria can be an inherent feature of this class of models. A major theme in this literature is also the relative value of research

[1]Outside of endogenous growth literature, Hovsepian (2017) studies a stylized model where investors allocate their asset between a clean and a dirty sector that are subject to strategic complementarity and specialization externality. She shows that if the effect of strategic complementarity is stronger than that of the specialization externality, multiple equilibria exist.

[2]Lemoine (2022) studies complementarity between resource use and innovation, but restricts forward-looking innovation incentives. While we have a simpler resource use structure, we can focus on the role of expectations, thus making our papers complementary.

subsidies and pollution taxes (Hart, 2019); we show that the balance between the two shifts because of the need for a coordination device.

Finally, by reconciling the observation of continued innovation in polluting technologies with the expectation of an eventual switch to clean technologies, our paper contributes to the understanding of asset stranding (van der Ploeg and Rezai, 2020) and the transition risk (Bolton and Kacperczyk, 2021). Our results suggest that continued investment in polluting technology is not necessarily a reflection of firms being myopic, but could be exactly due to the importance attributed to future profitability. Our results also point to the limitation of a single price instrument: even when environmental policy imposes a carbon tax that reflects future marginal damages or when markets price in a carbon risk premium, the economy may still coordinate on a suboptimal transition path.

The rest of the paper is organized as follows: Section 2 presents the model, and Section 3 solves the decentralized equilibrium. Section 4 shows that multiple steady states and transitional paths exist. Section 5 solves the planner's problem and analyzes the implication of multiple equilibria for climate policy. Finally, Section 6 concludes.

2 The model

We adopt the Acemoglu et al. (2012) model in a continuous time setting with two major modifications. Contrary to the original model, we allow labor to be mobile between production and research and adopt an infinite patent length instead of one-period patents. In the following, time subscripts are omitted whenever no confusion would arise.

2.1 Final goods producers

There are two final goods sectors in the economy, clean (Y_c) and dirty (Y_d). The final goods are produced by competitive producers using labor and a continuum of sector-specific intermediates following a Cobb-Douglas technology:

$$Y_j = L_j^{1-\alpha} \int_0^1 q_{ji} x_{ji}^{\alpha} di, \qquad j \in \{c, d\} \tag{1}$$

where L_j is the production labor hired in sector j, q_{ji} and x_{ji} are the quality and quantity of the intermediate good i in sector j. Final goods producers are price takers in both the final goods market and the factor markets. They decide on the factor demand (L_j, $\{x_{ji}\}_{i=0}^{1}$) to maximize their profit $\pi_j = P_j Y_j - wL_j - \int_0^1 P_{ji} x_{ji} di$, where P_j is the price of the final good j, w is wage, and P_{ji} is the price of the intermedaite good i in sector j.

While the clean sector is carbon free, the production of the dirty sector generates carbon emission according to

$$E = a_d Y_d, \tag{2}$$

where a_d is the emission intensity.

2.2 Intermediate goods producers

Each intermediate good x_{ji} is produced by a monopolist using the final goods of that sector. The unit cost of production increases with its quality q_{ji} so that one unit of intermediate good requires q_{ji} units of final goods j. Monopolists hire research labor s_{ji} for R&D to improve the quality of their products according to:[3]

$$\dot{q_{ji}} = \mu Q_j s_{ji}, \tag{3}$$

where μ is a research productivity shifter and $Q_j \equiv \int_0^1 q_{ji} di$ is the sector-wide average quality level. The latter proxies for the knowledge stock[4] on which innovation builds: the more knowledge accumulated for a class of technology, the more there is to draw on in R&D.[5]

The intermediate goods monopolists choose the amount of production x_{ji} and the level of research effort s_{ji} to maximize the net present value of its profits $\pi_{ji} = x_{ji}(P_{ji} - P_j q_{ji})$, subject to the demand for their goods as derived from the factor

[3] We choose to model innovation as the result of inhouse R&D à la Smulders and Nooij (2003), rather than "creative destruction". The former gives simpler mathematical expressions and seems to be at least equally empirically relevant as is shown recently by Garcia-Macia et al. (2019). This modeling choice does not alter the qualitative results in this paper.

[4] We use "knowledge" and "technology" interchangeably throughout.

[5] In Appendix OA6 we show that positive spillovers in innovation are not crucial for our main result of self-fulfilling prophecies; even negative spillovers (e.g. when Q_j in (3) gets a negative exponent) are compatible with self-fulfilling prophecies, as long as they are not too strong and are offset by complementarities in production.

demand by the final goods producers.

2.3 Households

The representative household derives utility from a composite consumption good, C, which is a CES aggregate of two substitutable sectoral goods, clean (C_c) and dirty (C_d):

$$C = \left[C_c^{\frac{\sigma-1}{\sigma}} + C_d^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where σ is the elasticity of substitution between the two sectoral goods. Consumers care about not only aggregate consumption but also climate damage according to the following instantaneous utility function

$$U = \ln\left[(1 - D(S))C\right], \tag{5}$$

where the damage ratio $D(S)$ is an increasing function of the carbon stock S.

Households supply inelastically one unit of labor at wage w, invest in assets V with return r, and discount future utility at the rate ρ. They maximize life-time utility $W_0 = \int_0^\infty \ln[1 - D(S)]C(t)e^{-\rho t}dt$ subject to the intertemporal budget constraint $\dot{V} = rV + w - P_c C_c - P_d C_d$, while taking the climate damage ratio $D(S)$ as given.

2.4 Climate dynamics and damage

Following Golosov et al. (2014), we assume $1 - D(S)$ to be

$$1 - D(S) = \exp\left(-\gamma(S - \bar{S})\right), \tag{6}$$

where $\gamma > 0$ is a damage parameter and $\bar{S}$ is the pre-industrial carbon concentration.

Carbon concentration, in turn, increases with the emission from the dirty output production and is given by

$$S_t = \int_{-\infty}^{t} \left[\phi_L + \phi_D e^{-\delta(t-s)}\right] E_s ds, \tag{7}$$

where ϕ_L is the share of the carbon emission that stays in the atmosphere forever, $\phi_D/(1 - \phi_L)$ is the share from the remaining emission that does not exit the atmosphere immediately but rather decays gradually, and δ is the geometric decay rate.

3 Direction of technical change in decentralized equilibrium

We derive the equilibrium conditions and show that investment complementarities drive self-fulfilling prophecies in the innovation decision. To highlight that this finding is independent of the savings decision, we defer the dynamic equilibrium analysis of the labour market to Section 4.

3.1 Equilibrium conditions

Producers Profit maximization of the final goods producers gives the usual factor demand, for labor and intermediates, respectively:

$$w = (1-\alpha)P_j \frac{Y_j}{L_j}, \tag{8}$$

$$P_{ji} = \alpha P_j L_j^{1-\alpha} q_{ji} x_{ji}^{\alpha-1}. \tag{9}$$

Intermediate goods monopolists maximize the net present value of their profits subject to the demand for their goods give by (9), leading to the following first order conditions (see Appendix OA1 for detailed derivation):

$$P_{ji} = \frac{1}{\alpha} P_j q_{ji}, \tag{10}$$

$$\mu Q_j \lambda_{ji} \leq w \perp s_{ji} \geq 0, \tag{11}$$

$$\dot{\lambda}_{ji} = r\lambda_{ji} - \frac{\partial \pi_{ji}}{\partial q_{ji}}, \tag{12}$$

where λ_{ji} denotes the firm's shadow value of quality improvements. Equation (10) is the usual markup rule. In (11), $\mu Q_j \lambda_{ji}$ represents the contribution of a marginal unit of research to the present value of the firm's future profit, while wage w captures the marginal cost of research effort. This equation thus determines the monopolist's investment decision, equating marginal benefits and costs in case of active research. Finally, (12) is the arbitrage equation that determines the shadow value of quality improvements.

Combining demand (9) and markup rule (10), we conclude that all firms in the same sector sell the same equilibrium quantity. Next, calculating their profits we find that these are linear in their own good's quality and that all monopolists enjoy the same marginal profit from quality improvement $\partial\pi_{ji}/\partial q_{ji}$. Thus, from (12), they face the same shadow value of innovation λ_{ji}; we henceforward write λ_j. The forward-looking component of a firm's investment decision thus depends on sector-level variables only. We can characterize the production side of the economy as:

$$Y_j = \alpha^{\frac{2\alpha}{1-\alpha}} Q_j L_j, \tag{13}$$

$$\dot{Q}_j = \mu s_j Q_j, \tag{14}$$

$$\pi_{ji} = (1-\alpha)\alpha P_j Y_j \frac{q_{ji}}{Q_j}, \tag{15}$$

where $s_j \equiv \int_0^1 s_{ji} di$ is the sectoral research input. According to (13) and (14), sectoral production reduces to a Ricardian production function in which productivity is proportional to the aggregate technology stock Q_j, which grows with aggregate research effort in that sector. All firms in the sector sell the same quantity, but firms with higher quality can sell at a higher price, (10), and thus make higher profits. Since final goods firms spend a constant fraction on intermediate goods and intermediate goods producers have a constant markup, total profits are a constant share $(1-\alpha)\alpha$ of revenue, while an individual firm's share in total profits is q_{ji}/Q_j according to (15).

Households Utility maximization of the households leads to the usual static demand functions and the Euler equation:

$$\frac{C_c}{C_d} = \left(\frac{P_c}{P_d}\right)^{-\sigma}, \tag{16}$$

$$r = \rho + \hat{C} + \hat{P}, \tag{17}$$

where P is the price index of consumption defined by $PC = P_c C_c + P_d C_d$. Equation (16) shows that relative demand responds to relative price with elasticity σ. (17) shows that households require a rate of return on their savings that reflects their impatience (ρ), a premium for postponing consumption ($\hat{C}$), and for inflation ($\hat{P}$).

Market clearing Goods market clearing requires that in each sector total production net of intermediate input use equals consumption. Using equilibrium quantities of x_{ji} and Y_j (see (13)), we find

$$C_j = Y_j - \int_{i=0}^{1} q_{ji} x_{ji} di = (1-\alpha^2) Y_j. \tag{18}$$

Labor market clearing requires that total (exogenous) supply equals demand for production and research:

$$1 = L + s_c + s_d, \tag{19}$$

where $L \equiv L_c + L_d$ is the total production labor.

Static equilibrium The allocation of profits, production labor, and consumer spending over the clean and dirty sectors depend on the pre-determined (i.e. history dependent) technology stocks Q_c and Q_d. We define θ_j to reflect the relative importance of a given technology stock Q_j as:

$$\theta_j \equiv \frac{Q_j^{\sigma-1}}{Q_c^{\sigma-1} + Q_d^{\sigma-1}}. \tag{20}$$

From equations (8), (13), (16), and (18), we find

$$\frac{L_c}{L_d} = \frac{P_c C_c}{P_d C_d} = \left(\frac{Q_c}{Q_d}\right)^{\sigma-1} = \frac{\theta_c}{\theta_d}, \tag{21}$$

Since $\theta_c + \theta_d = 1$, in equilibrium θ_j equals sector j's labor and spending shares:

$$\theta_j = \frac{L_j}{L} = \frac{P_j C_j}{PC}. \tag{22}$$

3.2 Investment complementarities

From (15), profits of intermediate goods firms are related to spending shares:

$$\pi_{ji} = \frac{q_{ji}}{Q_j}(1-\alpha)\alpha\theta_j PY,$$

which shows that (i) total profits in sector j are proportional to spending $\theta_j PY$ and (ii) firms earn a share in total profits that reflects their relative quality q_{ji}/Q_j. Since profits are linear in own quality, marginal profits depend on sectoral variables only: they are proportional to θ_j/Q_j. Using (20) and (21), we express relative marginal profits as:

$$\frac{\partial \pi_{ci}/\partial q_{ci}}{\partial \pi_{di}/\partial q_{di}} = \left(\frac{Q_c}{Q_d}\right)^{\sigma-2}. \tag{23}$$

Investment shifts spending shares and profits. A firm investing in higher quality directly experiences higher profits (through own quality q_{ji}), but non-investing firms are affected as well (through Q_j). Higher quality in sector j shifts expenditure to this sector as long as $\sigma > 1$ and the sector as a whole reaps more profits (demand shift effect). Non-investing firms see their share in these profits fall as their quality is relatively lower (business stealing effect). From (23) it is clear that the demand shift effect dominates the business stealing effect if and only if $\sigma > 2$, so that innovations by firms within the same sector become strategic complements: investment by any firm increases the profits from investment for all other firms in the sector.

In Appendix OA6, we generalize the model to allow for more sources of complementarities. In particular, we show that complementarities become stronger with learning-by-doing spillovers in production (parameterized by ε) and with the share of own sector inputs in the production of intermediates (parameterized by $\omega \in [0,1]$). The condition for strategic complementarity is then $\sigma > 1 + (1 - \omega\alpha)/((1-\alpha)(1+\varepsilon))$, with our model being a special case with $\varepsilon = 0$ and $\omega = 1$.

3.3 Coordination in the static setting: a thought experiment

As is well known, strategic complementarities may lead to coordination problems. This is immediate in a static setting. As a thought experiment, suppose our model economy exists only for one period. Innovation decision then becomes a static two-stage coordination game. From (3), innovation effort s_{ji} determines individual and sectoral technology stocks according to $q_{ji} = q_{ji0} + \mu s_{ji} Q_{j0}$ and $Q_j = (1+\mu s_j) Q_{j0}$. The marginal benefit of innovation for an individual firm is then given by $\partial \pi_{ji}/\partial s_{ji} = (\partial \pi_{ji}/\partial q_{ji}) \mu Q_{j0}$, while the marginal cost of innovation, the wage paid to scientists, is the same for all firms. From (23), the relative benefit of innovating in the clean sector is thus $(Q_c/Q_d)^{\sigma-2}(Q_{c0}/Q_{d0})$, while the relative cost is 1. If all firms together invest in clean only, i.e. if $s_c = s, s_d = 0, Q_c = Q_{c0}(1+\mu s), Q_d = Q_{d0}$, the relative benefit of clean innovation is $(1+\mu s)^{\sigma-2}(Q_{c0}/Q_{d0})^{\sigma-1}$; this investment is an equilibrium if this expression exceeds the unitary relative cost (and is a stable equilibrium if $\sigma > 2$). Similarly, all firms investing in dirty is a stable equilibrium if $(1+\mu s)^{\sigma-2}(Q_{d0}/Q_{c0})^{\sigma-1} > 1$. Hence if $(1+\mu s)^{-(\sigma-2)} < (Q_{c0}/Q_{d0})^{\sigma-1} < (1+\mu s)^{\sigma-2}$, both equilibria exist.[6] In other words, if the economy starts with the initial quality levels not too far apart, two stable equilibria exist. Following Krugman we call this range of initial conditions for which multiple equilibria exist the "overlap".

The same logic holds in Acemoglu et al. (2012), since the innovation decision in their model has the same two-stage feature with a similar cost-and-benefit structure to ours due to one-period patents and a similar production structure. Innovators only care about immediate profits and the economy evolves recursively. However, Acemoglu et al. (2012) did not point out the complementarities. Nor did they analyse the multiplicity. In the next section we show how the coordination issue arises in a fully dynamic setting.

3.4 Multiple rational expectations equilibria

Back to our infinite horizon model, firms' innovation decisions are driven by the contribution of higher quality to *all* future profits, λ_j. Together with the research productivity, μQ_j, it determines the value of innovation for future profits in sector j. We define the following measure of future market conditions in sector j:

$$m_j \equiv \frac{\mu Q_j \lambda_j}{\mu Q_c \lambda_c + \mu Q_d \lambda_d} = \frac{Q_j \lambda_j}{Q_c \lambda_c + Q_d \lambda_d}, \tag{24}$$

where $\mu Q_j \lambda_j$ is a sector-j firm's marginal value of innovation (see (11)) and m_j represents the (marginal) market valuation share of investing in that sector. The forward-looking variable m_c reflects how promising it currently is to invest in future green market rather than the other market.

Equation (11) shows that innovation will not take place in the sector in which one unit of research labor generates the lowest expected value: if $Q_c \lambda_c > Q_d \lambda_d$ the dirty sector cannot innovate and the clean sector innovates if the wage is not too high. Similarly, if $Q_c \lambda_c < Q_d \lambda_d$ only the dirty sector can innovate. Expressed in

[6] A third (unstable interior) equilibrium exists with some firms investing in the clean sector and other investing in the dirty sector.

terms of the market valuation share, this condition reads:

$$m_j > 1/2 \;\Leftrightarrow\; \hat{Q}_j \geq \hat{Q}_{-j} = 0. \tag{25}$$

We use hats to denote growth rates, $\hat{x} \equiv \dot{x}/x$ for any x. Intuitively, innovation only occurs in the sector that has the highest value of innovation. This inequality delineates three innovation regimes: if $m_c > 1/2$ the economy is in the *clean-only innovation regime* with $s_c > 0$ and $s_d = 0$; if $m_c < 1/2$ the economy is in the *dirty-only innovation regime* with $s_c = 0$ and $s_d > 0$; the *simultaneous-innovation regime* with $s_c > 0$ and $s_d > 0$ requires $m_c = 1/2$.[7] Market valuation share m_c is a continuous variable that cannot jump (unless unexpected shocks arise). Hence, if it starts above (or below) $1/2$ and the economy is in the clean-only (or in the dirty-only) regime, it remains so for some time. For consistency, we thus also require the simultaneous regime to be one where research is active in both sectors for a non-degenerate period of time. That is, the simultaneous regime is one where not only $m_c = 1/2$ is required but also $\dot{m}_c = 0$.

Under rational expectations, the forward looking valuation share m_c is connected to future development of market share θ_c, as it drives profits. The arbitrage equation (12) establishes this connection and implies:[8]

$$m_j > \theta_j \;\Leftrightarrow\; \hat{\lambda}_j > \hat{\lambda}_{-j}. \tag{26}$$

Intuitively, if the market valuation share of a sector is larger than its current market share, investors anticipate a growing market for that sector and the value of quality improvement grows faster in that sector than in the other sector.

To see how the direction of technical change evolves over time, we time differentiate (20) and (24):

$$\dot{\theta}_c = \theta_c(1-\theta_c)(\sigma-1)(\hat{Q}_c - \hat{Q}_d). \tag{27}$$

$$\dot{m}_c = m_c(1-m_c)\left(\hat{\lambda}_c - \hat{\lambda}_d + \hat{Q}_c - \hat{Q}_d\right) \tag{28}$$

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[7]We do not consider the stagnant case with $s_c = s_d = 0$ as a separate innovation regime, as it can be written as an equilibrium with $L = 1$ in any of the three innovation regimes.

[8]Denote marginal profits by $\bar{\pi}_{ji} \equiv \partial \pi_{ji}/\partial q_{ji}$ and relative variables by hats, $x_r \equiv x_c/x_d$ for any x. Then from (23) and (24), we find $\bar{\pi}_r/\lambda_r = \theta_r/Q_r\lambda_r = \theta_r/m_r = (\theta_c - m_c)/\theta_d m_c + 1$. From (12) we find $\hat{\lambda}_c > \hat{\lambda}_d \Leftrightarrow \bar{\pi}_r/\lambda_r < 1$. Combining with the previous result we find $\hat{\lambda}_c > \hat{\lambda}_d \Leftrightarrow m_c > \theta_c$.

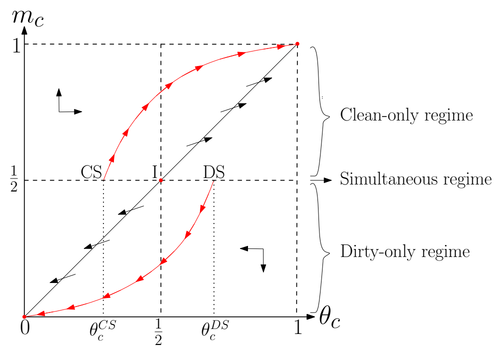

Figure 1: The (θ_c, m_c) equilibrium dynamics for $\sigma > 2$, showing the direction of technical change (small arrows) and the overlap $[\theta_c^{CS}, \theta_c^{DS}]$.

Equations (25)-(28) summarize the investment block of the model with the exception that the total research effort (i.e. the speed of innovation), $1-L$, is yet unknown, which depends on labour market equilibrium and the savings decision. Nonetheless, (25)-(28) allow us to analyse the direction of technical change independently of the rest of the model. Intuitively, the *relative* reward of investment pins down the direction of technical change for any speed of innovation.

We define a candidate steady state as an equilibrium in which θ_c and m_c are constant or asymptotically constant. They are steady states of the model conditional on yet undetermined savings decision $(1-L)$.[9] From (25)-(28), the phase diagram in Figure 1 and Proposition 1 follow.[10] The proofs for this and all subsequent propositions and lemmas are provided in Appendix A.

Proposition 1. *Suppose $\sigma > 1$. There exist three candidate steady states: a clean candidate steady state with $m_c = \theta_c = 1$ and innovation only in the clean sector, a dirty candidate steady state with $m_c = \theta_c = 0$ and innovation only in the dirty sector, and an (unstable) interior candidate steady state with $m_c = \theta_c = 1/2$ and innovation in both sectors. If and only if $\sigma > 2$, there exists a range of initial clean market share $\theta_c(0)$, for which both the clean and dirty candidate steady states can be reached in a rational expectations equilibrium.*

[9] Since (27) and (28) are part of the full model, any steady state of the full model must be a steady state of the subsystem while the reverse is not necessarily true.

[10] Figure 1 is a two-dimensional projection of the full phase diagram, which is essentially the full phase diagram conditional on the savings decision $(1-L)$.

In Figure 1, the horizontal line $m_c = 1/2$ is the regime border between clean (North) and dirty innovation (South) only. On the line innovation can be simultaneously clean and dirty. The three candidate steady states are illustrated by the three red dots in Figure 1. As detailed in the proof, the red path in the upper (lower) part of the figure illustrates an equilibrium path towards the clean (dirty) candidate steady state. The clean (dirty) path intersects with the regime border at θ_c^{CS} (θ_c^{DS}) and the clean (dirty) candidate steady state can be reached whenever the historically inherited market share starts above θ_c^{CS} (below θ_c^{DS}).

If $\sigma > 2$, the equilibrium paths towards the two corner candidate steady state overlap for a non-degenerate region of the initial market share $\theta_c(0)$. The overlap region ($[\theta_c^{CS}, \theta_c^{DS}]$ in Figure 1) gives the range of initial clean market share such that both corner candidate steady state can be reached. Within the overlap, future market size justifies current investment. With high substitution between the two goods, consumers relatively easily substitute towards the good that becomes relatively cheap. If the dirty sector invests, prices fall and consumers shift spending to the sector. Thus, future market size is big for these goods, and innovators realize a higher return on innovation in this market, thus further lowering prices and reinforcing the incentive for consumers to mainly spend on these goods. The spending share of the sector keeps increasing so that ultimately spending on dirty goods completely dominates the market. This explains why the economy can move to the dirty steady state. Similar arguments hold for the clean steady state.

The investment complementarity is independent of the knowledge spillovers implied by (3). Appendix OA6 shows that the overlap remains when knowledge spillovers are muted or when intersectoral knowledge spillovers are introduced. The reason is that research productivity is only affected by knowledge stock accumulated up to that point. In other words, the *knowledge spillover externality* affects the value of investment through the inherited knowledge stock. Since this stock is predetermined, knowledge spillover from current investment by rival firms cannot affect one's value of investment and is not a source of investment complementarity in the market.[11] In contrast, through the *demand externality* total investment affects

[11] Including direct spillovers from rivals' contemporaneous innovation effort would introduce an additional - rather obvious - source of complementarity.

future profits and thus the value of investment.

The implication of Proposition 1 is that investors' beliefs matter. If market shares are within the overlap, innovators' beliefs about future profitability determine which of the multiple rational equilibria is chosen. If the current green share is small but still within the overlap and firms are optimistic about future green market size, they invest and their beliefs come true - a self-fulfilling prophecy. But if all firms have pessimistic beliefs about a green transition, all firms invest in brown and their beliefs are also rational.

4 Self-fulfilling prophecies in the market economy

4.1 General equilibrium

We now analyse the full dynamics of the model by including labor market equilibrium and households' savings decisions which together determine the allocation of labor over production and research.

We denote any sector with active research by k, that is, $s_k > 0$. Combining (11), (12), (15), (13), and (8), we find

$$\hat{\lambda}_k = r - \alpha\mu L_k \tag{29}$$

$$r = \alpha\mu L_k + \hat{w} - \hat{Q}_k, \tag{30}$$

The rate of return that active innovators can offer is expressed in (30). The rate of return that investors require is expressed in (17). Capital market clearing follows from equating (30) and (17). Since final goods consumption expenditure is proportional to wage cost in the production sector, we can rewrite (17) as $r = \rho + \hat{L} + \hat{w}$, which combined with (30) and (22) characterizes capital market equilibrium:

$$\dot{L} = L\left[\alpha\mu\theta_k L - \hat{Q}_k - \rho\right]. \tag{31}$$

This general equilibrium version of the Ramsey rule shows that the more the discount rate falls short of the real rate of return on innovation, the more is the equilibrium production of consumption goods postponed to the future (i.e. $\dot{L}/L > 0$). The return on innovation is increasing in the market size $\theta_k L$, markup rate α, and the R&D productivity shifter μ. The increase in R&D productivity over time, as driven by $\hat{Q}_k > 0$, instead make postponement of R&D attractive and reduce $\dot{L}/L > 0$.

Given the identity of the research active sector, $\hat{Q}_j$ and $\hat{\lambda}_j$ in (27), (28), and (31) can be expressed in terms of L and θ_c. These three equations thus characterize the dynamic equilibrium in terms of the three endogenous variables θ_c, m_c, and L. The following lemma shows the resulting dynamic system.

Lemma 1. *The dynamic equilibrium is captured by the evolution of the clean market share θ_c, production labor L, and market valuation share m_c:*

$$\dot{\theta}_c = (\sigma - 1)\theta_c(1-\theta_c)\mu(s_c - s_d) \tag{32}$$

$$\dot{L} = L[\alpha\mu\theta_k L - \mu s_k - \rho] \tag{33}$$

$$\dot{m}_c = \alpha\mu L m_k(m_c - \theta_c) + \mu(s_c - s_d)m_c(1-m_c), \tag{34}$$

where k denotes the research-active sector. In the clean-only regime ($k = c$): $m_c > 1/2$, $s_c = 1-L$, and $s_d = 0$; in the dirty-only regime ($k = d$): $m_c < 1/2$, $m_k = 1-m_c$, $\theta_k = 1-\theta_c$, $s_c = 0$, and $s_d = 1-L$; and in the simultaneous regime ($k = c,d$): $m_c = 1/2$, $s_c = (1-L)/2 + \alpha L(\theta_c - 1/2)$, and $s_d = 1-L-s_c$. Active innovation in both sectors in the simultaneous regime further requires

$$\frac{1}{2}\left(1 - \frac{1-L}{\alpha L}\right) \le \theta_c \le \frac{1}{2}\left(1 + \frac{1-L}{\alpha L}\right). \tag{35}$$

Lemma 1 makes clear that within each innovation regime, $\dot{\theta}_c$ and $\dot{L}$ do not depend on m_c. We can thus analyze the dynamics of the spending share θ_c and the product market size L in two-dimensional phase diagrams, as illustrated in Figure 2. In the simultaneous regime (panel b), the dotted lines mark the active-research condition (35). The phase diagrams in Figure 2 depict changes in the speed of technical change $(1-L)$ as the market share θ_c evolves over time. The direction of technical change continues to be determined by changes in the market valuation share m_c. From Lemma 1, $\dot{m}_c$ depends on all three variables. As a two-dimensional projection, Figure 1 illustrates the direction of technical change conditional on L.

We define a steady state (an asymptotic steady state) as an equilibrium in which θ_c, m_c and L are constant (asymptotically constant). We indicate (asymptotic) steady state values with an asterisk. The phase diagrams in Figures 1 and 2 allow us to identify the steady states as the intersection of the zero-motion loci.

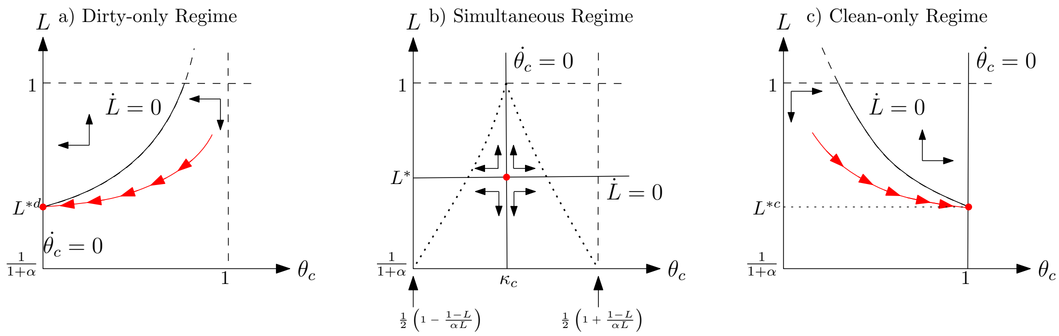

Figure 2: The (θ_c, L) phase diagrams, showing equilibrium total investment $s = 1 - L$ (red arrows or dot) for each regime.

Proposition 2. *Suppose $\sigma > 1$ and $\rho < \alpha\mu/2$. The decentralized economy has three steady states: (1) an unstable interior steady state with simultaneous R&D in both sectors, where $m_c^* = \theta_c^* = 1/2$, and $L^* = (1+\alpha)^{-1}(1+2\rho/\mu)$; (2) a saddlepath stable, asymptotic steady state with innovation in the dirty sector only, where $m_c \to m_c^{*d} = 0$, $\theta_c \to \theta_c^{*d} = 0$, $L \to L^{*d} = (1+\alpha)^{-1}(1+\rho/\mu)$; and (3) a saddlepath stable, asymptotic steady state with innovation in the clean sector only, where $m_c \to m_c^{*c} = 1$, $\theta_c \to \theta_c^{*c} = 1$, $L \to L^{*c} = (1+\alpha)^{-1}(1+\rho/\mu)$.*

The three steady states are illustrated by the red dots in Figure 2. The clean (dirty) steady state can only be reached from the clean-innovation (dirty-innovation) regime. Within the clean- or dirty-innovation regime, the red lines with arrows illustrate the unique path towards each of the two corner steady states. These paths are the projection of the three-dimensional saddlepaths on the (θ_c, L) plane, just as the red lines in Figure 1 depict the projection on the (θ_c, m_c) plane.

Proposition 2 focuses on the case where innovation is active (that is, $L < 1$) in all steady states. This requires $\rho < \alpha\mu$ for the corner steady states and $\rho < \alpha\mu/2$ for the interior steady state.

4.2 Self-fulfilling prophecies

To analyse the transition to the steady states, we first define equilibrium paths.

Definition 1 (Equilibrium Path)**.** *An equilibrium path is a sequence of $(\theta_{c,t}$, m_{ct}, L_t, s_{ct}, $s_{dt})$ that satisfies (32), (33), and (34) at any point in time, and approaches one* *of the two saddlepath-stable steady states as $t \to \infty$.*

An equilibrium path defined above is a sequence of $(\theta_{c,t}, m_{ct}, L_t, s_{ct}, s_{dt})$ that jointly maximizes firm profits and household lifetime utility, while clearing the factor, goods, and capital markets at the same time. Put in the context of rational expectations, an equilibrium path is the outcome of coordinated beliefs among all agents about all future actions of other agents and the resulting state of the economy. As agents are symmetric and atomistic, in equilibrium all agents must share the same belief, which can be represented by the equilibrium sequence of $(\theta_{c,t}, m_{ct}, L_t, s_{ct}, s_{dt})$. When agents coordinate on such a belief, it will turn out to be consistent with the optimizing behavior of all agents and market clearing of all markets at any point in time and thus consistent with their belief. Multiplicity arises if for the same initial condition multiple such equilibrium paths exist. In this case, the economy is free to select any such path, meaning that agents can coordinate on any such beliefs that will turn out to be rational. In our deterministic setting of the model, once a selection is made at time 0, there is no more uncertainty and rational expectations collapse into perfect foresight.

We further refer to any equilibrium path that approaches the clean steady state a clean path, and one that approaches the dirty steady state a dirty path.

Clean vs dirty path Proposition 2 confirms that the three candidate steady states listed in Proposition 1 are indeed steady states of the full model. As a result, the complementarity condition in Proposition 1 applies. Under complementarity ($\sigma > 2$), a range of initial conditions (“overlap") exists for which both the clean and dirty steady states can be reached.

We conclude that coordination of beliefs determines to which of the two steady states the economy evolves if it starts within the overlap. The full dynamic representation in Lemma 1 solves for the transition path and size of the overlap, as numerically illustrated in Figures 3-4 and analytically characterized in the following proposition:

Proposition 3. *Suppose $\sigma > 1$ and $\rho < \alpha\mu/2$. If and only if $\sigma > 2$, a range of initial states exists from which both corner steady states can be reached through* *a rational expectations equilibrium path. This range ("overlap") increases with substitution σ and decreases with impatience ρ.*

The bigger the elasticity of substitution, the more consumers shift expenditure to the innovating sector and increase profits in this sector. Expectations of a bigger future market with associated investment in this market thus become more quickly justified. This explains why the overlap increases with the elasticity of substitution σ. An increase in impatience has the opposite effect: it reduces the present value of future profits and lowers investment incentives, thus decreasing the speed at which either sector can gain market size. If investors are infinitely impatient, only current profit matters and the investment becomes a static coordination game as in Section 3.2. In Appendix OA6.3, we further show that an increase in patent length has a similar effect to lowering impatience, as both increase the time horizon of investors.

Fast vs delayed transition Figure 1 shows that when tracing the saddlepathh from a corner steady state backward in time, the path eventually intersects with the regime border. At that point a regime switch must occur, which is also associated with the switch in direction of change for the spending share θ_c. This implies that when a saddlepath crosses the regime border, there are spending share levels that the saddlepath towards the same steady state visits multiple times. Thus, for some initial spending share, there are multiple equilibrium paths reaching the same steady state but each corresponding to a different length of transition time.

Since each such starting point is associated with a different future market condition $m_c(0)$, the selection again depends on firms' belief of future profitability and thus the innovation decision of other firms in the economy. If the market expects ultimately a clean steady state, no matter in which innovation regime the economy starts, it must enter the clean-only regime before reaching the steady state. That is, the only rational expectation about a clean steady state is that, in the long run, all innovation will occur in the clean sector. For the short and medium run, however, if firms expect a speedy transition, it is only rational to invest only in the clean sector; if firms expect a delayed transition, it can be rational to temporarily invest in the dirty sector. Since firms' innovation decisions are strategic complements, such beliefs about the speed of transition can also be self-fulfilling.

Proposition 4. *From an initial* $\theta_{c,0} \in \left[\theta_c^{CS}, \theta_c^{DS}\right]$*, there are multiple transition paths towards each corner steady state. In particular,*

1. *a fast transition, in which the economy selects immediately the corner stable path, is always possible;*
2. *delays in transition before the economy finally selects the corner stable path are also possible; further, there exist* $\bar{\sigma}$ *and* $\bar{\bar{\sigma}}$*, where* $2 < \bar{\sigma} < \bar{\bar{\sigma}}$*, such that*
   - *(a) if* $2 < \sigma < \bar{\sigma}$*, temporary simultaneous R&D is the only possible delay;*
   - *(b) if* $\sigma \in [\bar{\sigma}, \bar{\bar{\sigma}})$*, delay must include temporary regime switches between the two corner innovation regimes;*
   - *(c) if* $\sigma > \bar{\bar{\sigma}}$*, delay must include temporary stagnation with no R&D.*

The importance of σ for the pattern of delays again reflects the role of market responsiveness. For a relatively low σ, relative demand responds to innovation-induced changes in relative price only sluggishly. No matter which sector is expected to eventually dominate, changes towards this sector can occur only slowly. It takes a lot of cumulative changes to finally tip the balance towards one of the two sectors decisively. As σ becomes higher, the speed of demand responses picks up. An initial bias of expectations in favor of one of the two sectors can now be much more easily confirmed by market responses, and become self-reinforcing. This higher speed of demand response, however, also adds to volatility, as it becomes easier to tip the balance in favor of a specific sector.

The implication of Proposition 4 is that even if firms believe that the economy will eventually transition to a clean steady state, how fast this transition happens is subject to coordination. If firms believe that there will be some period of indecisiveness concerning the relative profitability, it can indeed be rational to innovate only in the dirty sector temporarily. We thus offer a *forward-looking* explanation why we may observe predominantly dirty innovation, even if firms believe that the economy will switch to clean technologies in some future time.

Numerical example To provide a graphical illustration of the overlap and the different transition scenarios, we now consider a numerical example. We set the labor share in production to the common value of two-third so that $\alpha = 1/3$ and the long-run growth rate to 1.25 percent. We allow the time preference rate ρ and the

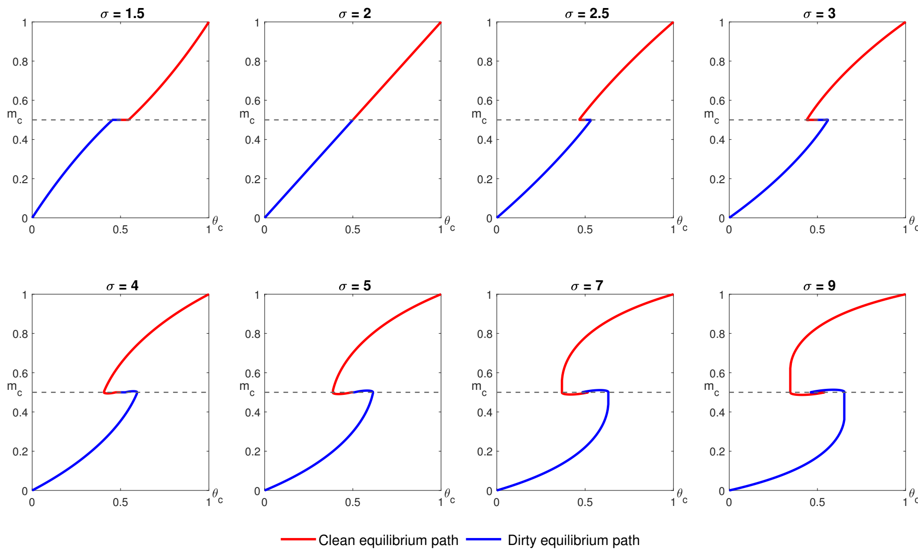

Figure 3: Equilibrium paths with different σ values

elasticity of substitution σ to vary. Given ρ, we derive μ from the long-run growth rate $g = \mu(1 - L_k) = (\alpha\mu - \rho)/(1 + \alpha)$. For the baseline time preference rate of $\rho = 0.01$, $\mu = 0.08$ and $\rho < \alpha\mu/2$ holds.

Figure 3 shows the projection of the equilibrium paths on the (θ_c, m_c) plane. As in Proposition 3, the clean and dirty equilibrium paths overlap for all σ values larger than 2. Further, the overlap region grows larger with increasing σ. In terms of the possible delays, for $\sigma = 2.5$ and $\sigma = 3$, the delays involve simultaneous R&D, as illustrated by the flat part of the equilibrium paths, where $m_c = 1/2$. With σ between 4 and 7, the equilibrium paths involve regime switches between the clean-only and dirty-only regimes. Finally, with $\sigma = 9$, the equilibrium paths contain vertical sections corresponding to temporary stagnation, where $\dot{\theta}_c = 0$. Figure 4 shows that, as in Proposition 3, the overlap region becomes smaller with larger ρ.

Figure 4 shows the projection of the equilibrium paths for different time preference rates. As in Proposition 3, the larger the time preference rate, the smaller is the size of the overlap.

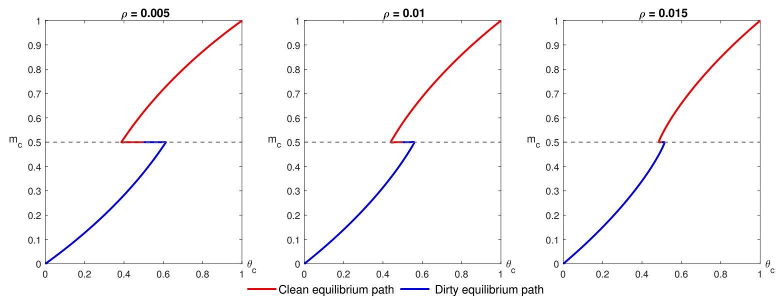

Figure 4: Equilibrium paths with different ρ values ($\sigma = 3$)

5 Self-fulfilling prophecies and optimal policy

The previous section has shown that investment complementarities lead to self-fulfilling prophecies in the market economy, both in terms of the long-run equilibrium and the transition path. We now characterize the social optimum and study the implications of coordination failure for optimal policy.

5.1 Social optimum

The planning problem is equivalent to maximizing (5) subject to (4), (1), (18), (19), (3), the equation of motion for carbon concentration $\dot{S}$ (derived from time differentiating (7)), and the non-negativity constraints of L_j and s_{ji} for all j and all i. The detailed solution to the planning problem is provided in Appendix OA3.

For the static optimum, the social planner corrects both the misallocation caused by the market power of the intermediate monopolists and that caused by the climate externality. Let τ_d^s denote the marginal cost of carbon emissions in terms of the dirty goods. The planner's sectoral output is given by

$$Y_c = \alpha^{\frac{\alpha}{1-\alpha}} Q_c L_c, \quad Y_d = ((1-\tau_d^s)\alpha)^{\frac{\alpha}{1-\alpha}} Q_d L_d. \tag{36}$$

The marginal cost of carbon emissions equals $\tau_d^s \equiv a_d \frac{\partial \dot{S}}{\partial E} \lambda_S / \zeta_d$, i.e. the social cost of the carbon stock in utility terms, λ_S, corrected for the effect of dirty production on the carbon stock $a_d \frac{\partial \dot{S}}{\partial E}$ and for the utility price of the dirty goods, $\zeta_d = (C_d/C)^{-1/\sigma}/C$. We show in Appendix OA3 that $\frac{\partial \dot{S}}{\partial E} \lambda_S$ is a constant equal to $\Phi \equiv \phi_L \gamma / \rho + \phi_D \gamma / (\rho + \delta)$, while ζ_d changes over time. In particular, the emissions cost τ_d^s decreases in the clean technology stock Q_c, and increases in the dirty technology stock Q_d, production labor L, and the social cost of carbon per unit of dirty good $a_d\Phi$. Intuitively, if the economy grows bigger through investment in the dirty sector, polluting goods become more abundant relative to environmental quality and the social cost of emissions grows. However, if the economy grows bigger through clean investment, dirty goods become scarce both relative to clean goods and relative to environmental quality. The willingness to give up dirty goods in exchange for environmental quality then falls.

Comparing (36) to (13), we see that the market under-provides goods because of monopolistic markup α. Also, the market provides too much dirty goods because it does not price carbon emission. While the planner adjusts output in both sectors upward to correct the underproduction caused by monopoly power, she also recognizes that the production of the dirty goods causes emission and increases climate damage. The latter leads to a downward adjustment of dirty production.

For the dynamic optimum, the planner's shadow price for the technology stock q_{ji}, denoted by λ_j^s,[12] evolves according to

$$\hat{\lambda}_k^s = \rho - \mu(L_k + s_k) \tag{37}$$

where k denotes the research active sector (that is, $s_k > 0$). Comparing (37) to (29), we see that the market's shadow value of investment is the net present value of $\alpha\mu L_k$, while the planner's is the net present value of $\mu(L_k + s_k)$.[13] Since the former is smaller than the latter due to monopoly pricing and the knowledge spillovers, the market puts too little value on innovation.

The rest of the optimal conditions are similar to the decentralized equilibrium conditions, and the planner's solution can be characterized in a few key variables.

Let θ_c continue to be the relative technology of the clean sector (see (20)), and L continue to denote the total production labor. Further, similar to m_j in the decen-

[12] Since all firms within the same sector face the same marginal benefit and cost for innovation, the shadow price for q_{ji} is again the same for all producers in the same sector. We thus drop the producer index i. The superscript s distinguish the planner's shadow price from that of the market.

[13] To allow comparison between market and planner's shadow prices, we express them both in utility terms, which requires equating the market consumer price level to marginal utility, $P = 1/C$. From (17) we get $r = \rho$.

tralized equilibrium, denote by

$$m_j^s \equiv \frac{Q_j \lambda_j^s}{Q_c \lambda_c^s + Q_d \lambda_d^s}, \tag{38}$$

the share of the clean sector in the total market valuation in the planner's solution. The dynamics of the planner's solution can be characterized in terms of variables θ_c, Q_d, L, and m_c^s. Whereas in the unregulated market economy dynamics only θ_c, L, and m_c matter, in the optimal dynamics Q_d matters in addition since the dirty technology stock drives costly climate damages. The differential equation system of the four variables is provided in Appendix OA3. We indicate optimal (asymptotic) steady-state values by the superscript "$**$". The next proposition characterizes the long run social optimum.

Proposition 5. *Suppose $\sigma > 1$. The welfare maximizing path converges to an asymptotic steady state with innovation in the clean sector only, $s_d = 0$, with $m_c^s \to 1$, $\theta_c \to 1$, and $L \to \rho/\mu$; the steady state dirty technology stock $Q_d^{**} \geq Q_{d0}$ and asymptotic steady state emission cost $\tau_d^{s**} \in [0,1]$ both depend on initial conditions, where the latter is a positive function of the former as implicitly defined by*

$$\frac{\tau_d^{s**}}{(1-\tau_d^{s**})^{1/(1-\alpha)}} = \left(\frac{\alpha^{\alpha/(1-\alpha)}(1-\alpha) a_d \Phi \rho}{\mu} \right) Q_d^{**}. \tag{39}$$

Not surprisingly, welfare maximization requires a transition to a clean steady state. Because production in the dirty sector increases the atmospheric carbon stock, the climate costs in a growing economy with a non-vanishing dirty sector become infinitely large. As long as the clean sector has relatively low quality, substituting clean for dirty consumption entails a short-run cost, but in the long run this cost will always be dwarfed by the climate cost of a dirty-innovation-based growth path.[14]

Since in the long run the clean sector provides all goods in the economy and all growth stems from this sector, the productivity level in the dirty goods sector becomes asymptotically irrelevant. Depending on the starting level of the dirty asset stock and on whether in the short run some dirty innovation takes place, different (but bounded) levels of dirty technology stock are compatible with a clean steady

[14] Technically, a dirty steady state violates the transversality condition for the carbon stock.

state, i.e. there is hysteresis with repect to dirty assets and emission cost. This implies stranded assets: ultimately there is no role for dirty technology in the optimal steady state, so that all initial investments become stranded in the long run.

5.2 Self-fulfilling prophecies under a Pigouvian carbon tax

The optimal transition path is characterized by a sequence of decisions with regards to which sector should be active in research. Conditional on this sequence of choices concerning the research-active sector, the decentralized economy only faces three standard types of inefficiency – monopoly pricing, knowledge spillover, and climate externality – and the social optimum can be implemented in the market economy through regulation that directly address the three externalities.

Proposition 6. *Suppose $\sigma > 1$ and the regulated market economy innovates in the sector that the planner would choose given state variables. The social optimum can then be implemented by the simultaneous use of the following policies:*

1. *an optimal industry policy consisting of a revenue subsidy $\tau_\alpha = \alpha^{-1} - 1$ and a sector-specific technology subsidy $\tau_{qj} = s_j w / Q_j$ (per unit of q_{ji}) for the intermediate goods monopolists, and*
2. *an optimal climate policy consisting of a Pigouvian carbon tax τ_d per unit of dirty output revenue, which is set to equal the social cost of emissions in terms of dirty goods, τ_d^s, at all times.*

The above proposition shows that when the coordination failures concerning the choice of the research-active sector are somehow solved, the social optimum can be easily implemented in the decentralized economy by a first-best policy consisting of an industry and a climate policy. However, the next proposition shows that given such policies, the optimal steady state is not the only steady state.

Proposition 7. *Suppose $\sigma > 1$. Under the industry and climate policies in Prop 6:*

1. *the regulated market economy converges to an asymptotic steady state with innovation in the clean sector only, $s_d = 0$, with $m_c \to 1$, $\theta_c \to 1$, and $L \to \rho/\mu$; (39) holds; both the steady-state dirty technology stock $Q_d^* \geq Q_{d0}$ and asymptotic steady-state emission cost $\tau_d^* \in [0, 1]$ depend on initial conditions and possibly on expectations.*

Parameter	ρ	α	μ	σ	$\theta_{c,0}$	$Q_{d,0}$	a_d	γ	ϕ_L	ϕ_D	δ	S_0	$\bar{S}$
Value	0.01	1/3	0.08	1.5	0.177	24k	0.198	0.0002	0.2	0.32	0.002	877	581

Table 1: Parameter values

2. *given $(Q_{c,0}, Q_{d,0})$, there exist $\underline{Q}_c(Q_{d,0}) > 0$ and $\bar{Q}_c(Q_{d,0}) \geq \underline{Q}_c(Q_{d,0})$, and if the initial clean technology level is*
   (a) *high such that $Q_{c,0} \geq \bar{Q}_c(Q_{d,0})$, equilibrium is unique and identical to the social optimum; along the transition, innovation occurs only in the clean sector;*
   (b) *medium such that $Q_{c,0} \in [\underline{Q}_c(Q_{d,0}), \bar{Q}_c(Q_{d,0}))$, multiple equilibria exist; there exist a path where research is only active in the clean sector and at least one other path where research is active temporarily only in the dirty sector before switching permanently to the clean sector;*
   (c) *low such that $Q_{c,0} < \underline{Q}_c(Q_{d,0})$, all paths must include temporary dirty-only research, before research switches to the clean sector permanently.*

Proposition 7 shows that, given gross substitutability between the clean and dirty goods, there exists a region of initial conditions for which multiple equilibria are possible under a Pigouvian carbon tax.[15] To provide a graphical illustration of the multiple equilibria and their welfare ranking, we consider a numerical example. We use the same parameter values for ρ, α and μ as in Section 4 and set $\sigma = 1.5$. We set $\theta_{c,0}$ to the 2019 renewable energy share in global final energy consumption (17.7 percent, IEA, 2022). Using this and the 2019 world GDP per capita (\$11,019 constant 2015 USD, World Bank, 2022b), we calibrate the initial dirty output $Y_{d,0}$ to \$6,143. Assuming a 5 percent research labor share (i.e. $L_0 = 0.95$), the initial dirty technology stock $Q_{d,0}$ is calibrated to \$23.57k. Using $Y_{d,0}$ and the 2019 per capita carbon emission of 1.22 metric tons (World Bank, 2022a), the emission intensity a_d is calibrated to 0.198 tonnes of carbon per thousand USD. The population size is assumed to be constant at the 2019 level. For the damage function and the carbon

[15]While in the unregulated market economy $\sigma > 2$ is required for self-fulfilling prophecies, in the regulated economy $\sigma > 1$ suffices as the optimal industry policy in Prop 6 internalizes the social value of innovation. In addition, the size of the overlap also increases when optimal industry policy is applied. See Figure 9 in Appendix OA3.

dynamics we follow the calibration of Golosov et al. (2014), where the values of ϕ_L, ϕ_D, and δ are adjusted to annual frequency, and S_0 is updated to account for carbon emissions of recent years. An overview of the parameters is provided in Table 1.

Using the above parameter values, we solve the model numerically to find all equilibrium paths for a given initial condition $(Q_{d,0}, \theta_{c,0}, S_0)$, and compare their welfare. Using the same $(Q_{d,0}, S_0)$ values as in Table 1 while varying $\theta_{c,0}$, Figure 5 illustrates the welfare comparison of all equilibrium paths for each $\theta_{c,0}$. In particular, the red curve represents the welfare for a clean-only equilibrium path, the dashed magenta line represent an equilibrium path with temporary dirty research, and the dotted blue line represents another equilibrium path with even more dirty research. We see that the entire θ_c range can be divided into three regions as stated in Proposition 7, where multiple equilibria exist in the mid range of $\theta_{c,0}$. Whenever multiple equilibria exist, the clean-only path is the path with the highest welfare. Thus, unless all equilibrium paths feature temporary dirty research (such as in the leftmost region in Figure 5), the planner will prefer the clean-only path.

Intuitively, if the economy starts with enough clean capital, the planner prefers to concentrate all future innovation in the clean sector. Then complementarities in the clean sector and the need to make a transition to a clean steady state benefit clean innovation over dirty innovation from the start. If, however, the economy starts with relatively little clean assets, complementarities make innovation in the dirty sector relatively attractive in the short run, even though in the long run the economy will transition to a clean economy and assets in the dirty sector will be ultimately stranded. In this case, the cost of more asset-stranding in the long run is outweighed by the short-run benefit of larger consumption, and the planner will temporarily allocate all research to the dirty sector.

Notice from Figure 5 that $\theta_{c,0} = 0.177$ falls within the region of multiple equilibria. Indeed, we can find three different equilibrium paths corresponding to different levels of long-run dirty technology stock, as illustrated in Figure 6. Compared to the equilibrium path with clean research only, a path with temporary dirty research will take much longer to reach the steady state. Investing in the dirty capital thus leads to a delayed transition, and the more dirty capital the economy accumulates, the longer is the delay and the larger the stock of stranded assets.

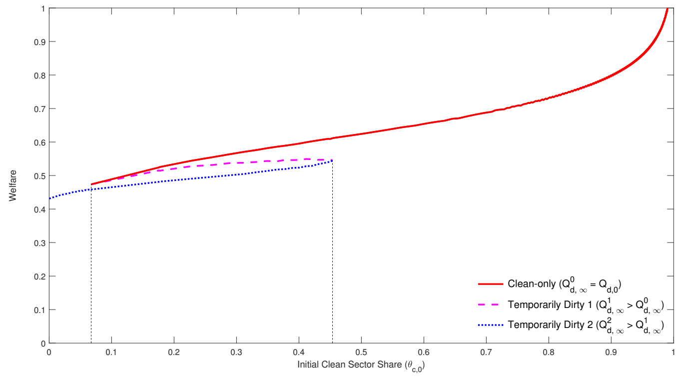

Figure 5: Welfare comparison of different equilibrium paths

Figure 6 further gives us a hint why a delayed transition may be individually rational for innovators as an alternative for the equilibrium without delay. Even though the long-run tax rate is very high in a delayed transition, the tax rate starts low and needs to first increase over time, before declining to its long-run level. In comparison, a fast transition requires a higher tax at the beginning. Thus, even if agents expect the long-run climate policy to be stringent, they may expect it to start loose and only become stringent slowly. In the meantime (when tax is still low), agents "exploit" the complementarities to coordinate on dirty research. A low emission tax raises the return to innovation in the dirty sector and if labor shifts from production to research this justifies the expectation of a low tax (as the Pigouvian tax increases in the scale of production[16]) – a self-fullfilling prophecy arises.

To understand why two equilibria with delayed transition exist – one with a long period of dirty innovation and the other with a short one – consider the following two opposite forces. Shortening the delay reduces the long-run cost of emissions, but also reduces the complementarities from which investors benefit. In an equilibrium with a small delay in clean innovation, dirty innovation becomes profitable despite little complementarities because the modest long-run tax allows dirty technologies to be used more intensively. In an equilibrium with a long delay in clean innovation, there are prolonged short-run gains from complementarities, but dirty technologies are more quickly phased out in the longer run due to the higher tax.

[16] See (OA.24) in the appendix: for given initial condition θ_{c0} and Q_{d0}, the tax can only be lower in the equilibrium with delay, relative to the value in the equilibrium without delay, if L is lower.

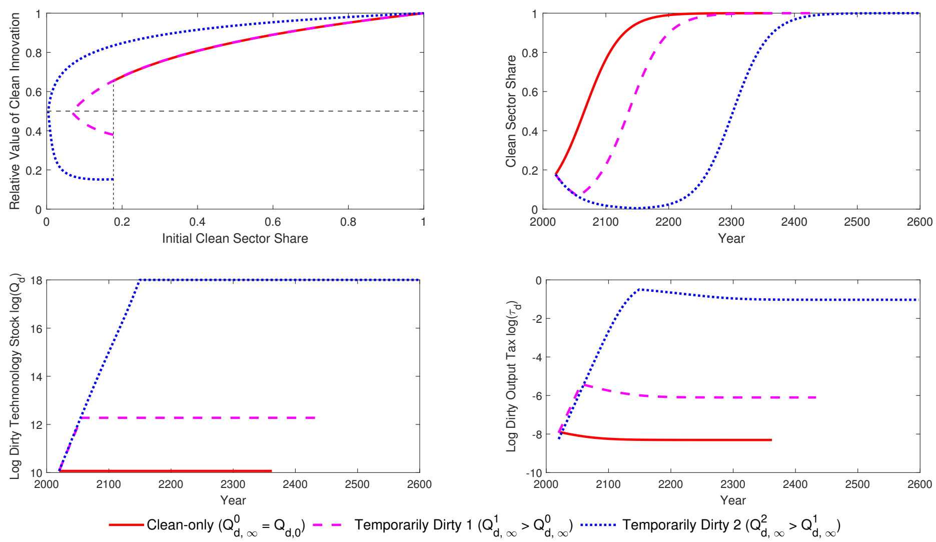

Figure 6: Fast vs delayed transitions

From Proposition 7 and the numerical example, we see that although the presence of a Pigouvian carbon tax on the dirty sector effectively rules out the dirty steady state, it does not pin down a unique transition path towards a clean steady state. Whether there will be a fast transition or rather a delayed transition with a larger than optimal amount of stranded asset is again subject to coordination, and again, self-fulfilling prophecies may arise.

5.3 The role of commitment

Propositions 6 and 7 presume that policymakers have information on firms' investment when setting the Pigouvian tax at each point in time. This is akin to assuming policymakers cannot commit to future policies and chooses the tax after investors have chosen investment. If this tax policy were replaced by a cap and trade policy and there were still no commitment, the same results hold since tax and quantity instruments are equivalent if policymakers move after the uncertainty regarding the cost of pollution is resolved. This is similar in Biais and Landier (2022), who find in the context of a two-stage sequential technology spillover model that a quantity instrument cannot solve the coordination problem if the regulator cannot commit.

Commitment does not solve the coordination failure if policymakers commits to future tax rates. The reason is that under commitment the tax path cannot change and the private sector takes future taxes as given. The complementarity in investment decisions is preserved and the coordination failure still arises. Nevertheless the tax path can affect the size of the overlap and the equilibrium pollution path.

Commitment can solve the coordination problem in our model if policymakers can commit to a quantity policy. If it fixes the allowable pollution per period, it fixes the quantity of output in the dirty sector and fully controls relative profitability. The quantity policy mutes the investment complementarity: due to the Leontief production structure (fixed emissions per unit of output), investment in higher productivity no longer pays if the quantity policy is binding. However, under less extreme assumptions (abatement possible in the dirty sector, or pollution in both sectors), the coordination failure is likely to return even under a committed quantity policy.

5.4 Coordination device

In the face of possible coordination failure, it may appear tempting to suggest a higher carbon tax to discourage agents delaying the transition. This requires the carbon tax to be higher than the Pigouvian level in the delayed paths. However, as we see from Figure 6, the carbon tax in a delayed transition rises sharply and becomes quickly much higher than that of the optimal path. Thus, a sustained high carbon tax would be necessary, with excessive stringency when the fast path is selected.

At the root of this climate policy dilemma is the lack of policy instruments that address the coordination failure per se. The economy is facing both a climate externality that is generated by dirty production, and an innovation coordination issue that is caused by strategic complementarity in firms' innovation decisions. While the two are linked through the use of dirty technology, they are ultimately two separate issues and a carbon tax cannot be expected to optimally deal with both issues. Instead, a coordination device becomes necessary.

Discussion from the previous section reveals that, when emissions are proportional to dirty output, a cap-and-trade policy under commitment may serve as coordination device. Regulatory standards that prescribe a maximum emission intensity for the final goods have a similar effect. By directly limiting the market share of dirty goods, regulatory standards can break complementarity. If policymakers can commit to a path of rising emission standards, this can similarly solve the coordination problem.

Another policy that has received lots of attention in the literature is R&D subsidy for clean research. From Proposition 7 we see that there exists a unique-equilibrium threshold of the initial clean technology level, $\bar{Q}_c(Q_{d,0})$. If the R&D subsidy can change this threshold to be below the initial clean technology level, then multiple equilibria can also be avoided.

Suppose policymakers can commit to a constant R&D subsidy rate of τ_{rd} per scientist hired by the clean sector for as long as $Q_{ct} \geq \bar{Q}_c(Q_{d,0})$. This lowers the unit wage cost of clean research to $(1-\tau_{rd})w$ and the innovation regime border shifts downwards:

$$m_j > (1-\tau_{rd})/(2-\tau_{rd}) \;\Leftrightarrow\; \hat{Q}_c \geq \hat{Q}_d = 0. \tag{40}$$

As the regime border affects the level of the unique-equilibrium threshold (see proof in Appendix A), we can re-write this threshold as $\bar{Q}_c(Q_{d,0}, \tau_{rd})$. The next proposition suggests that it is possible to lower the unique-equilibrium threshold to below the initial clean technology level by imposing a high enough clean R&D subsidy.

Proposition 8. *Suppose $\sigma > 1$. Given $(Q_{c,0}, Q_{d,0})$ and the industry and climate policies specified in Proposition 6, a temporary R&D subsidy on clean research can rule out multiple equilibria. The clean R&D subsidy must be sufficiently high such that $\tau_{rd} > \tilde{\tau}_{rd}$ such that the initial clean technology level is above the unique-equilibrium threshold ($Q_{c,0} > \bar{Q}_c(Q_{d,0}, \tilde{\tau}_{rd})$). The minimum subsidy rate $\tilde{\tau}_{rd}$ decreases with $Q_{c,0}$. Along the transition, research is only active in the clean sector.*

This shows that a temporary subsidy on clean research can indeed be used as a coordination device, as long as it is sufficiently large. Note that the application of this subsidy discourages dirty research altogether. On one hand, this suggests that, if the initial clean technology level is moderate ($Q_{c,0} \in [\underline{Q}_c(Q_{d,0}), \bar{Q}_c(Q_{d,0})]$), such a policy can coordinate agents on the optimal transition path and in addition has no fiscal impact. On the other hand, however, if the initial clean technology level is low ($Q_{c,0} < \bar{Q}_c(Q_{d,0})$), such a policy becomes distortionary as it causes the economy to forego temporary dirty research that is welfare-enhancing. Thus, although a clean R&D subsidy can serve as a coordination device, its use should be limited to situations where the initial clean technology level is not too low.

6 Conclusions

We have shown that self-fulfilling prophecies arise in a standard dynamic model of DTC, according to which the equilibrium direction of innovation is determined by expectations about the future path of innovation. If goods of different sectors are sufficiently good substitutes, a demand externality causes the investments in innovation by monopolistic suppliers within a sector to be strategic complements. Applying this finding to innovation directed towards clean versus polluting technologies, we have shown that the expectation of a delay in innovation in clean technologies provides the incentive for individual investors to delay their own innovation in this direction as well, thus justifying the expectation. An equilibrium without delay in clean innovation is also possible but requires that investors do not expect delay.

Our coordination failure model provides an explanation for persistence of innovation in pollution-intensive sectors and for the slow transition to a carbon-free economy. In our welfare analysis we found that the equilibria with delay feature excessive asset stranding, i.e. investment in assets that need to be phased out in the longer run, the cost of which would have been avoided in an equilibrium with faster transition.

The implications for environmental policy are striking. First, if knowledge spillovers – the standard feature of endogenous technology and DTC models – are internalized through generic innovation subsidies, inefficient delays in clean innovation are still possible and even magnified. If the pollution externalities are internalized through a Pigouvian pollution tax that reflect all (current and future) damages, inefficient delay and asset stranding still can occur even if all agents believe that only clean technology will be used in the long run. A standard Pigouvian tax on emission cannot serve as a coordination device and optimally tax carbon at the same time. We thus recommend the use of a separate policy for coordinating the transition path selection.

While our analysis focuses on the analytics of the dynamic coordination failure, the qualitative results can be expected to carry over to larger models that are more suitable for calibration and detailed policy analysis. For example, allowing for multiple sectors and several abatement options or including physical capital only affect the margins along which the economy responds to shifts in innovation direction and to environmental policy, but have no direct effect on the nature of the investment complementarities in innovation – the key driving force behind our results. Model extensions along these lines can help us evaluate the quantitative implications of the coordination failures and are an important direction for future research.

We have assumed rational expectation without stochastic and with perfect information, by only considering equilibrium paths in which expected time path exactly coincide with realized path. As is usual in rational-expectation models with multiple equilibria, the model is silent about where the expectations come from. Future work could introduce heterogeneity of (subjective) beliefs, noisy signals about outcomes and players' actions, and explicit expectation formation.

Our model features monopolist competition of a large number of small firms. Coordination is arguably easier in the presence of large players, whose action changes market conditions considerably. The role of large players in coordination problem has been studied in the context of currency crisis (Corsetti et al., 2004). Future research could similarly introduce large players into the directed technical change framework and study more targeted policies in the green transition.

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A Proofs

A.1 Proof Proposition 1

From (27) and (28), $\dot{\theta}_c = 0$ requires $\theta_c = 1$, $\theta_c = 0$, $\hat{Q}_c = \hat{Q}_d$, while $\dot{m}_c = 0$ requires $m_c = 1$, $m_c = 0$, or $\hat{\lambda}_c - \hat{\lambda}_d + \hat{Q}_c - \hat{Q}_d = 0$. Since $\theta_c = 1$ is only consistent with Q_c growing faster than Q_d, this candidate steady state must have $m_c = 1$. Since $m_c > 1/2$ so by (25) the economy is in the clean-only regime. Similarly, a candidate steady state with $\theta_c = 0$ must also have $m_c = 0$, and by (25) the economy is the dirty-only regime. A candidate steady state with $\hat{Q}_c = \hat{Q}_d$ can only be in the simultaneous regime, which requires $m_c = 1/2$. So $\dot{m}_c = 0$ can only hold if $\hat{\lambda}_c = \hat{\lambda}_d$, which by (26) means $m_c = \theta_c$. This proves the existence of the three candidate steady states.

We prove the rest of the proposition by constructing and examining the phase diagram (Figure 1). As per (25), the horizontal line $m_c = 1/2$ is the regime border between clean (North) and dirty innovation (South) only. On the line innovation can be simultaneously clean and dirty. As per (26), above the 45 degree line, $m_c > \theta_c$ and $\hat{\lambda}_c > \hat{\lambda}_d$. As per (27) and (28), this implies that in the North West part of the diagram both m_c and θ_c increase; in the South East part they both decline. On the 45-degree line, $m_c = \theta_c$ and by (26) we have $\dot{m}_c/\dot{\theta}_c = 1/(\sigma - 1)$. It follows that any equilibrium path can cross the 45-degree line at most once, since the path must have the same positive constant slope whenever it crosses the 45-degree line. This slope is flatter than (equal to, or steeper than) 1 if $\sigma > 2$ ($\sigma = 2$, or $\sigma \in (1,2)$).

We see immediately all three candidate steady states are on the 45-degree line. The interior candidate steady state cannot be reached from the clean-only regime, since this requires m_c to fall while θ_c to rise along the transition and thus the equilibrium path must come from the North West, which however contradicts our finding above that both θ_c and m_c must increase in this region. Similarly, the interior candidate steady state cannot be reached from the dirty-only regime. Thus it can only be reached from the simultaneous regime. However, there exists no equilibrium path towards it because such a path would have be a horizontal line which contradicts our finding that any path intersecting with the 45-degree line must have a positive slope if $\sigma > 1$. Thus, no equilibrium path can be found towards the interior candidate steady state so it is unstable as long as $\sigma > 1$.

Now consider the clean candidate steady state with $\theta_c = m_c = 1$. Any equilibrium path reaching this steady state must spend a non-degenerate amount of time in the clean-only regime, in which θ_c increases over time. Since an equilibrium path can only cross the 45-degree line once, tracing back in time from the clean candidate steady state any equilibrium path must remain above ($\sigma > 2$), below ($\sigma < 2$) or on ($\sigma = 2$) the 45-degree line until it hits the regime border at some market share θ_c^{CS}. There is thus at least one rational expectations equilibrium path to the clean steady state whenever the initial market share starts above this critical level, $\theta_c(0) \geq \theta_c^{CS}$.

Similarly, there is at least one rational expectations equilibrium path to the dirty steady state whenever $\theta_c(0) \leq \theta_c^{DS}$.

If $\sigma > 2$, any equilibrium path towards the clean candidate steady state is always above the 45-degree line as long as it remains in the clean-only regime (see the red path in the upper half of Figure 1). Thus, $\theta_c^{CS} < 1/2$. Similarly, $\theta_c^{DS} > 1/2$. Consequently, for any $\theta_c(0) \in [\theta_c^{CS}, \theta_c^{DS}]$, both candidate steady states can be reached.

If $\sigma = 2$, any equilibrium path towards the clean candidate steady state lies on the 45-degree line as long as it remains in the clean-innovation regime and $\theta_c^{CS} = 1/2$. Similarly, $\theta_c^{DS} = 1/2$. The interval $[\theta_c^{CS}, \theta_c^{DS}]$ degenerates into a single point where $m_c = \theta_c = 1/2$, which is the interior candidate steady state.

Finally, if $\sigma < 2$, any equilibrium path towards the clean candidate steady state is always below the 45-degree line as long as it remains in the clean-innovation regime. Thus, $\theta_c^{CS} > 1/2$. Similarly, $\theta_c^{DS} < 1/2$. There exists no $\theta_c(0)$ from which both candidate steady states can be reached.

A.2 Proof Lemma 1

For the clean-only and dirty-only regimes, $\hat{Q}_j$ ($j \in \{c, d\}$) follows from (14) and the definition of the regime. Plugging $\hat{Q}_j$ in (27) gives θ_c, and plugging $\hat{Q}_k$ and (20) in (31) gives $\dot{L}$. We find $\dot{m}_c$ by plugging $\hat{Q}_j$, (12) and (29) in (28).

In the simultaneous regime, $\dot{m} = 0$ by definition, while (30) holds for both $k = c$ and for $k = d$. Together with (28), this gives us $\hat{Q}_c - \hat{Q}_d = \hat{\lambda}_d - \hat{\lambda}_c = \alpha\mu L(\theta_c - \theta_d)$. Substituting in $\hat{Q}_c = \mu s_c$ and $\hat{Q}_d = \mu(1 - L - s_c)$, we find $s_c = (1-L)/2 + \alpha L(\theta_c - 1/2)$. Plugging this into (27) gives us $\dot{\theta}_c$. Setting $k = c$ in (31) and using the derived s_c expression, we find $\dot{L}$.

(35) follows from requiring $0 < s_c < 1 - L$ in the simultaneous regime.

A.3 Proof of Proposition 2

Prop 1 gives us the steady state values of θ_c and m_c. To find the steady state value of L, we set (33) to zero and plugging in the steady state value of θ_c.

Prop 1 already proves that the interior steady state is unstable. The stability of the corner steady states can be seen from the phase diagram in Figure 2. In the left panel, the direction of change of L and θ_c shows that the dirty steady state can only be approached from Northeast of the steady state and along the transition, both L and θ_c fall monotonically. There is thus a unique path towards the dirty steady state. Any path starting above the saddlepath implies that $L = 1$ in finite time, beyond which the arbitrage condition – i.e. the equation for $\dot{L}$ in Lemma 1 – can no longer be satisfied; any path starting below the path implies that L vanishes which is inconsistent with the transversality condition. Similar argument applies to the clean steady state.

A.4 Proof of Proposition 3

We have shown the condition for the overlap in Prop 1. For its size, we claim that $\partial\theta_c^{DS}/\partial\sigma > 0$ and $\partial\theta_c^{DS}/\partial\rho < 0$ while $\partial\theta_c^{CS}/\partial\sigma < 0$ and $\partial\theta_c^{DS}/\partial\rho < 0$. We prove here the claims for θ_c^{DS}. The proof for θ_c^{CS} is analogous.

Lemma 2. *In the dirty-only regime, for any $\theta_c > 0$, $\partial L/\partial\sigma < 0$ and $\partial L/\partial\rho > 0$.*

Proof of Lemma 2. In the θ_c-L plane (see Fig 7) the slope of the projection of the dirty equilibrium path in the dirty-only regime satisfies

$$\frac{\dot{L}}{\dot{\theta}_c} = \frac{L}{\sigma - 1}\left[\frac{(1+\rho/\mu) - (1+\alpha-\alpha\theta_c)L}{\theta_c(1-\theta_c)(1-L)}\right] \equiv f^{L\theta D}(\theta_c, L, m_c, \sigma, \rho). \qquad \text{(A.1)}$$

Evaluated at the same (θ_c, L) pair, this slope decreases with σ and increases with ρ since the term in square brackets is positive along the dirty-only equilibrium path.

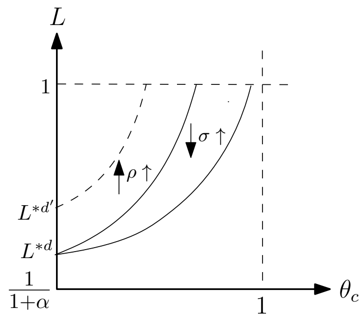

Figure 7: 2D projection in (θ_c, L)

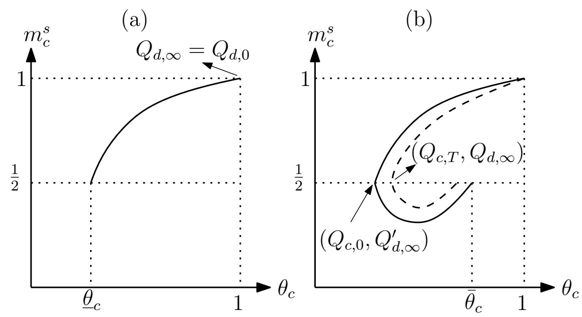

Figure 8: Thresholds for $Q_{c,0}$

Thus, equilibrium paths with different σ cross only once, namely at the steady state (since they share the same steady state) and the path becomes flatter with higher σ (see Figure 7). Equilibrium path with different ρ cannot cross, because a higher ρ corresponds to a higher steady state L so that a crossing requires the path with a higher ρ to have a flatter slope when crossing, a contradiction. Hence, the equilibrium path with a higher σ (or a lower ρ) has a lower L for any θ_c. $\square$

The projection of the equilibrium path in the dirty-only regime has the following slope in the θ_c-m_c plane:

$$\frac{\dot{m}_c}{\dot{\theta}_c} = \frac{1}{\sigma-1}\left(\frac{m_c(1-m_c)}{\theta_c(1-\theta_c)}\right)\left[1+\left(\frac{\alpha L}{1-L}\right)\frac{\theta_c - m_c}{m_c}\right] \equiv f^{m\theta D}(\theta_c, L, m_c, \sigma), \quad \text{(A.2)}$$

where $\theta_c > m_c$. Evaluated at the same (θ_c, m_c) pair, as long as $\sigma > 1$, this slope decreases with σ and increases with ρ since $1/(\sigma-1)$ and L decrease with σ, while L increases with ρ (by Lemma 2). Thus again, equilibrium paths with different σ or ρ can cross only once, namely at the steady state, and the path becomes flatter as σ becomes higher or ρ becomes lower. This proves $\partial\theta_c^{DS}/\partial\sigma > 0$ and $\partial\theta_c^{DS}/\partial\rho < 0$.

A.5 Proof of Proposition 4

We prove the possible delays for a transition towards the dirty steady state. The proof for delayed transition towards the clean steady state follows similar argument.

A regime switch must occur at $m_c = 1/2$. Denote the value of θ_c and L at the final regime switch towards the dirty-only regime by θ_c^{DS} and L^{DS}. The final regime-switching point is thus given by $(\theta_c^{DS}, L^{DS}, 1/2)$. If (35) holds at the final regime-switching point, the economy enters the dirty-only regime from the simultaneous regime; if (35) is violated but $L^{DS} < 1$, the economy enters from the clean-only regime; and if $L^{DS} = 1$, the economy enters through stagnation.

Note that θ^{DS} is the right boundary of the overlap in Prop 3 and is shown to increase with σ. We now show that L^{DS} is also increasing in σ.

Lemma 3. *$dL^{DS}(\sigma)/d\sigma > 0$.*

Proof of Lemma 3. Consider the projection of the dirty equilibrium path in the (m_c, L) plane within the dirty regime. From the phase diagram, we know $\dot{L}/\dot{m}_c > 0$. Further, the slope of the projection is given by

$$\frac{\dot{L}}{\dot{m}_c} = \frac{L}{1-m_c}\left[\frac{\alpha\theta_c L + \rho/\mu - [(1+\alpha)L-1]}{\alpha\theta_c L - m_c[(1+\alpha)L-1]}\right] \equiv f^{LmD}(\theta_c, L, m_c, \sigma). \tag{A.3}$$

Given (m_c, L), the slope f^{LmD} can be affected by σ through θ_c:

$$\frac{\partial f^{LmD}}{\partial \theta_c} = \frac{(1-m_c)\alpha L^2}{(-\dot{m}_c)^2}\left[(1-m_c)((1+\alpha)L-1) - \frac{\rho}{\mu}\right]. \tag{A.4}$$

Note that $\partial f^{LmD}/\partial\theta_c = 0$ if $L = \frac{1}{1+\alpha}\left[1 + \frac{\rho}{\mu(1-m_c)}\right] \equiv \check{L}(m_c)$. The slope of $\check{L}(m_c)$ is given by $f^{Lm} \equiv \partial\check{L}(m_c)/\partial m_c = (L - 1/(1+\alpha))/(1-m_c)$. From (A.3), whenever the projection of the equilibrium path and $\check{L}(m_c)$ intersect in the (m_c, L) plane, $f^{LmD} = L/(1-m_c) > f^{Lm}$. Thus, the projection of the equilibrium path and $\check{L}(m_c)$ intersect at most once. Since the two intersect at the dirty steady state, $L > \check{L}(m_c)$ holds for the all $m_c > 0$ along the equilibrium path. From (A.4), $\partial f^{LmD}/\partial\theta_c > 0$.

In addition, by Lemma 2, given L, a higher σ translates into a higher θ_c (see Figure 7). Thus, given (m_c, L), $\partial f^{LmD}/\partial\sigma = (\partial f^{LmD}/\partial\theta_c)(\partial\theta_c/\partial\sigma) > 0$. Consequently, the projections of equilibrium paths with different σ can at most intersect once in the (m_c, L) plane. Since they all intersect at the dirty steady state, it follows that for any $m_c > 0$, a higher σ corresponds to a larger equilibrium L. □

By Prop 3 and Lemma 3, both θ^{DS} and L^{DS} increase with σ. However, since within the dirty-only regime $\dot{L}/\dot{\theta}_c \to +\infty$ as $\theta_c \to 1$, the equilibrium path must reach $L = 1$ before reaching $\theta_c = 1$. Thus, there must exist $\bar{\bar{\sigma}} \in (2, \infty)$ such that $L^{DS}(\sigma) < 1$ for all $\sigma < \bar{\bar{\sigma}}$, and $L^{DS}(\sigma) = 1$ for all $\sigma \geq \bar{\bar{\sigma}}$.

At $\theta_c = \theta_c^{DS}$, the existence of the simultaneous regime requires (cf (35)):[17]

$$L \leq \frac{1-1/2}{1-1/2+\alpha(\theta_c^{DS}-1/2)} \equiv L^{RBS}(\theta_c^{DS}(\sigma)) = L^{RBS}(\sigma) \tag{A.5}$$

which is indirectly a function of σ through θ_c^{DS}.

If $\sigma = \bar{\bar{\sigma}} > 2$, $L^{DS} = 1$ but $\theta^{DS} > 1/2$ and $L^{RBS} < 1$ so that $L^{DS} > L^{RBS}$. If $\sigma = 2$, $L^{DS} < 1$ but $\theta_c^{DS} = 1/2$ and $L^{RBS} = 1$ so that $L^{DS} < L^{RBS}$. Since $\partial L^{RBS}(\sigma)/\partial\sigma < 0$ and $\partial L^{DS}/\partial\sigma > 0$, there exists a unique $\bar{\sigma} \in (2, \bar{\bar{\sigma}})$ such that $L^{DS}(\sigma) < L^{RBS}(\sigma)$ for all $\sigma \in (2, \bar{\sigma})$, and $L^{DS}(\sigma) \geq L^{RBS}(\sigma)$ for all $\sigma \geq \bar{\sigma}$.

If $\sigma \in (1, \bar{\sigma})$, $L^{DS}(\sigma) < L^{RBS}(\sigma)$ so that the economy reaches the final regime switch from the simultaneous regime; if $\bar{\sigma}^D \leq \sigma < \bar{\bar{\sigma}}$, $L^{DS}(\sigma) \geq L^{RBS}(\sigma)$ and $L^{DS} < 1$ so that the economy enters from the clean-only regime; finally, for all $\sigma \geq \bar{\bar{\sigma}}$ we have $L^{DS} = 1$ and thus there must be a stagnant period with no growth.

Finally, the next lemma shows that the simultaneous regime cannot be reached from a different regime so that for $\sigma \in (1, \bar{\sigma}^D)$, the simultaneous regime is the only possible delay. This completes the proof.

Lemma 4. *If the economy is not in the simultaneous regime at time t and no unexpected shocks occur, it cannot be in the simultaneous regime after t.*

Proof of Lemma 4. Suppose at time T the economy enters the simultaneous regime from the clean-only regime. We must then have $\lim_{t\to T} \dot{m}_c < 0$ and $\lim_{t\to T} m_c = 1/2$. From (OA.4) this requires $\theta_c > \left(1 + \frac{1-L}{\alpha L}\frac{\mu}{\mu}\right) 1/2$, but this violates the second inequality in (35). The proof for the case starting with dirty only is similar. □

A.6 Proof of Proposition 5

In this proof we also refer to the dynamic equilibrium conditions provided in Appendix OA3. A steady state in the social planner's solution requires

$$\dot{\theta}_{k,\infty} = 0, \;\; \dot{L}_\infty = 0, \;\; \dot{m}^s_{k,\infty} = 0, \;\; \dot{\tau}^s_{d,\infty} = 0; \tag{A.6}$$

$$\lim_{t\to\infty} \lambda^s_{j,t} q_{j,i,t} e^{-\rho t} = 0 \quad (j \in \{c,d\}); \qquad \lim_{t\to\infty} \lambda_{S,t} S_t e^{-\rho t} = 0. \tag{A.7}$$

(A.6) defines the steady state, and (A.7) is the TVCs concerning individual firm's technology stock and the aggregate carbon stock.

[17]Note that with $\theta_c^{DS} \geq 1/2$, only the second inequality in (35) is relevant.

In the clean-only regime, $\dot{m}^s_{c,\infty} = 0$ requires $m^s_{c,\infty} = \chi_{L,c,\infty} > 1/2$. This, together with $\dot{\theta}_{c,\infty} = 0$ and $\dot{\tau}^s_{d,\infty} = 0$, requires that $\theta_{c,\infty} = 1$, which by (OA.22) leads to $\chi_{C,c,\infty} = \chi_{L,c,\infty} = 1$. $\dot{L}_\infty$ then requires $L_\infty = \rho/\mu$, while $\dot{\tau}^s_{d,\infty} = 0$ is satisfied for any $\tau^s_{d,\infty} \in [0,1]$. From (OA.23) and the steady state values $\chi_{L,c} = 1 - \chi_{L,d} = 1$ and $L = \rho/\mu$, we find (39). It is easily verified that this solution also satisfies (A.7).

In the dirty-only regime, (A.6) can only be jointly satisfied if $\theta_{c,\infty} = 0$, $\chi_{C,c,\infty} = 0$, $\chi_{L,c,\infty} = 0$, $m^s_{c,\infty} = 0$, $\tau^s_{d,\infty} = 1$, and $L_\infty = 0$. But this means that $\hat{\lambda}_d + \hat{Q}_d = \rho - \mu L_d \to \rho$ and $\lim_{t\to\infty} e^{-\rho t}\lambda_{d,t}Q_{d,t}$ is a positive constant, which violates the TVC.

In the simultaneous regime, $m^s_c = \chi_{L,c} = 1/2$ holds. From (OA.26), (OA.27) and $\dot{\chi}_{L,c} = 0$, we have $\hat{\chi}_{C,c} \propto -\hat{\tau}^s_d$. For $\dot{\chi}_{C,c} = 0$, either $\hat{\chi}_{C,c} = \hat{\tau}^s_d = 0$ or $\chi_{C,c} = 0$. If $\hat{\tau}^s_d = 0$, from $\dot{L}_\infty = 0$, (OA.32) and (OA.33) we find $\hat{Q}_{d,\infty} = 0$, which violates $s_d > 0$. If $\chi_{C,c} = 0$, together with $\chi_{L,c} = 1/2$ we obtain $\tau^s_d = 1$, which combined with (OA.22) and $\chi_{L,c} = 1/2$ requires that $\hat{Q}_c < \hat{Q}_d$. However, from (OA.27) we have $\hat{\lambda} > 0$. From $\dot{L}_\infty = 0$, (OA.32), and (OA.33) we then find $\hat{Q}_d < 0$, which also violates $s_d > 0$.

A.7 Proof of Proposition 7

Under the policies specified in Prop 6, all equations of motion are the same for the regulated economy and the planner's solution (see Appendix OA3) so Prop 5 applies to the regulated economy. This proves part (1) of Prop 7.

The claims in part (2) follow directly from the existence and definitions of the two thresholds $\underline{Q}_c$ and $\bar{Q}_c$.

Lemma 5. *Denote by* $(Q_{c,T}, Q_{d,\infty})$ *the technology levels when the economy enters the clean regime for the final time.* $Q_{c,T}$ *is a non-monotonic function of* $Q_{d,\infty}$ *and there exists* $\bar{Q}_{c,1}$ *such that* $Q_{c,T} \leq \bar{Q}_{c,1}$ *for all* $Q_{d,\infty} \in (0,\infty)$.

Proof of Lemma 5. From (39), $\partial\tau_{d,\infty}/\partial Q_{d,\infty} > 0$ and each $Q_{d,\infty}$ corresponds to a unique $\tau_{d,\infty}$. Given $\tau_{d,\infty}$, the steady state and the equilibrium path are uniquely determined. The equilibrium path must have a left-most intersection with the $m_c = 1/2$ line (see Fig 8). Denote by $\underline{\theta}^l_c$ the relative technology level at this point. Given $Q_{d,\infty}$, there is a unique $\underline{\theta}^l_c$ and thus a unique $Q_{c,T}$. Thus, $Q_{c,T}$ is a function of $Q_{d,\infty}$.

Consider $Q_{d,\infty} \to 0$. In this case, $\tau_{d,\infty} \to 0$. At the limit, this is the case without a carbon tax and $\bar{\theta}_{c,1} \in (0,1/2)$. Denote $Q_{c,T}$ at this limit by $Q^0_{c,T}$. Clearly, $Q^0_{c,T} \approx 0$.

Now consider $Q_{d,\infty} \to \infty$. In this case, $\tau_{d,\infty} \to 1$. By the definition of $\chi_{L,c}$, $\chi_{L,c} \to 1$ for any positive θ_c that is not close to zero. Since in the clean regime $\dot{m}_c > 0$ and $m_c > \theta_c$ must hold, $m_c \to 1$ for any positive θ_c that is not close to zero. For the equilibrium path to intersect with the regime border, $\underline{\theta}_c^l \approx 0$ must hold. Since with $Q^0_{c,T}$ and $Q_{d,\infty}$, $\theta_c \approx 0$. It follows that $Q_{c,T} \approx Q^0_{c,T}$.

Since $Q_{c,T} = Q^0_{c,T}$ holds for both $Q_{d,T} \to 0$ and $Q_{d,T} \to \infty$, $Q_{c,T}$ is non-monotonic in $Q_{d,\infty}$. Setting $\bar{Q}_{c,1} \equiv \max\{Q_{c,T} | Q_{d,\infty} \in (0,\infty)\}$ completes the proof. □

Since m_c and θ_c are slow-moving variables, the clean steady state can only be reached from the clean-only regime. Any equilibrium path is thus either a clean-only path ($Q_{d,\infty} = Q_{d,0}$) or one with temporary dirty research ($Q_{d,\infty} > Q_{d,0}$).

Threshold $\underline{Q}_c$ We check if a clean-only path is possible for an initial condition. Such a path must lie entirely within the clean-only regime. By (39), $Q_{d,\infty} = Q_{d,0}$ pins down $\tau_{d,\infty}$ and a unique equilibrium path. The projection of this path on the (θ_c, m_c) plane must have a leftmost intersection with the $m_c = 1/2$ line (see panel (a) of Fig 8). Denote θ_c and Q_c at this intersection by $\underline{\theta}_c$ and $\underline{Q}_c$, respectively. Given $Q_{d,0}$, a clean-only path is only feasible if $\theta_{c,0} \geq \underline{\theta}_c$, or equivalently, if $Q_{c,0} \geq \underline{Q}_c$.

Threshold $\bar{Q}_c$ We check if equilibrium paths with temporary dirty research are possible. Such a path must have at least one regime switch and at the final regime switch, $Q_{c,T} \geq Q_{c,0}$ must hold ($=$ if only one regime switch).

(I) From Lemma 5, if $Q_{d,0}$ is very large such that $\partial Q_{c,T} / \partial Q_{d,\infty} < 0$, any equilibrium path with dirty research must have $Q_{c,T} < \underline{Q}_c$ (since $Q_{d,\infty} > Q_{d,0}$). Thus, if $Q_{c,0} \geq \underline{Q}_c$, clean-only path is the only equilibrium. In this case, $\bar{Q}_c = \underline{Q}_c$.

(II) Now consider the case when $Q_{d,0}$ is not too large so that $Q_{c,T}$ is non-monotonic in $Q_{d,\infty}$ for all $Q_{d,\infty} \geq Q_{d,0}$.

(IIa) If $Q_{c,0} \geq \bar{Q}_{c,1}$, by Lemma 5, no path with $Q_{c,T} \geq Q_{c,0}$ and $Q_{d,\infty} > Q_{d,0}$ can be found so there are no equilibria with temporary dirty research.

(IIb) If $Q_{c,0} < \bar{Q}_{c,1}$, by Lemma 5, at least one equilibrium path with $Q_{c,T} = Q_{c,0}$ and $Q_{d,\infty} > Q_{d,0}$ can be found. Such a path must also have a rightmost intersection $\bar{\theta}_c$ with the $m_c = 1/2$ line (see panel (b) of Fig 8). For this path to be feasible for an initial condition, $\theta_{c,0} \leq \bar{\theta}_c$ must hold. Given $Q_{d,0}$, this translates to an upper bound for $Q_{c,0}$. Consider all possible paths with one regime switch and let $\bar{Q}^2_{c,0}$ be the largest of all upper bounds associated with their rightmost intersection with the regime border. If $Q_{c,0} > \bar{Q}^2_{c,0}$, no equilibria with one regime switch exist.

(IIc) Similarly, consider all possible paths with more than one regime switches ($Q_{c,T} \in (Q_{c,0}, \bar{Q}_{c,1}]$ and $Q_{d,\infty} > Q_{d,0}$) and let $\bar{Q}^3_{c,0}$ be the largest of all Q_c upper bounds associated with the rightmost intersection of the paths with the regime border. If $Q_{c,0} > \bar{Q}^3_{c,0}$, no equilibria with more than one regime switch exist.

Summarizing (I) and (IIa)-(IIc), we conclude that no equilibria with temporary dirty research exist if $Q_{c,0} \geq \bar{Q}_c$, where $\bar{Q}_c = \underline{Q}_c$ if $Q_{d,0}$ is large (so that $\partial Q_{c,T}/\partial Q_{d,\infty} < 0$) and $\bar{Q}_c \equiv \min\{\bar{Q}_{c,1}, \max\{\bar{Q}_{c,2}, \bar{Q}_{c,3}\}\}$ otherwise.

A.8 Proof of Proposition 8

For any $Q_{d,\infty} \in (0,\infty)$, the left-most intersection of the equilibrium path and the regime border, $\underline{\theta}^l_c$, decreases as τ_{rd} increases and the regime border shifts down. Thus, also $Q_{c,T}$ becomes smaller. Since $Q_{c,T}$ decreases for all $Q_{d,\infty} \in (0,\infty)$, both $\underline{Q}_c$ and $\bar{Q}_{c,1}$ are decreasing in τ_{rd}. If $\bar{Q}_c = \underline{Q}_c$, $\partial \bar{Q}_c/\partial \tau_{rd} < 0$ clearly holds. If $\bar{Q}_c = \min\{\bar{Q}_{c,1}, \max\{\bar{Q}_{c,2}, \bar{Q}_{c,3}\}\}$, either $\max\{\bar{Q}_{c,2}, \bar{Q}_{c,3}\}$ also decreases with τ_{rd} so $\partial \bar{Q}_c/\partial \tau_{rd} < 0$, or as $\bar{Q}_{c,1}$ falls, eventually $\bar{Q}_c = \bar{Q}_{c,1}$ holds and thus $\partial \bar{Q}_c/\partial \tau_{rd} < 0$.

For $Q_{c,t} > \bar{Q}_c(Q_{d,0}, \tau_{rd})$ to hold at all time, τ_{rd} must be higher than the level given by $Q_{c,t} = \bar{Q}_c(Q_{d,0}, \tau_{rd})$. As $Q_{c,t}$ grows over time, $\tilde{\tau}_{rd}$ falls.

OA Online Appendix for “Self-fulfilling Prophecies in the Transition to Clean Technology”

OA1 Supply side equations in the decentralized equilibrium

Substituting the firm’s demand curve (9) and research technology (3), we write the Hamiltonian for the firm’s maximization problem as:

$$H_{ji} = \alpha P_j L_j^{1-\alpha} q_{ji} x_{ji}^{\alpha} - P_j q_{ji} x_{ji} - w s_{ji} + \lambda_{ji} \mu Q_j s_{ji}.$$

The first order condition (FOC) for s_{ji} is (11). The FOC for x_{ji} reads

$$\alpha P_j L_j^{1-\alpha} q_{ji} x_{ji}^{\alpha-1} \alpha = P_j q_{ji},$$

which after substitution of (9) gives (10) and

$$x_{ji} = \alpha^{\frac{2}{1-\alpha}} L_j. \tag{OA.1}$$

Substituting (10) and (OA.1) into the definition of profits gives (15). Substitution of (OA.1) into the production function for Y_j gives (13).

The FOC for q_{ji} gives (12). Since $\partial \pi_{ji} / \partial q_{ji}$ is the same across firms, λ_{ji} is also the same across firms. Omit the subscript i in (12), divide both sides by λ_j, and substitute (15), we find

$$\dot{\lambda}_j / \lambda_j \equiv \hat{\lambda}_j = r - (1-\alpha) \alpha P_j Y_j / (Q_j \lambda_j). \tag{OA.2}$$

Using (11), (8), and (13), we find (29) for the research active sector. From (11) we have $\hat{\lambda}_k = \hat{w} - \hat{Q}_k$, which combined with (29) gives (30).

Finally, for any sector j, (OA.2) can be rewritten as

$$\hat{\lambda}_j = r - \alpha \mu L_k \frac{P_j L_j / \lambda_j}{P_k L_k / \lambda_k}.$$

Using $P_c Q_c = P_d Q_d$ as derived from (8) and (13) and the definition of m_c, the above equation is equivalent to

$$\hat{\lambda}_j = r - \alpha \mu L_j \frac{Q_k \lambda_k}{Q_j \lambda_j} = r - \alpha \mu L_j \frac{m_k}{m_j}. \tag{OA.3}$$

OA2 Differential equations for the three innovation regimes

There are three innovation regimes. In the clean-only regime, $m_c > 1/2$, $s_d = 0$, $s_c = 1 - L$, and

$$\begin{cases} \dot{\theta}_c &= (\sigma - 1)\theta_c(1 - \theta_c)\mu(1 - L), \\ \dot{L} &= L\left[\mu L(1 + \alpha\theta_c) - (\mu + \rho)\right], \\ \dot{m}_c &= m_c\mu\left[\alpha L(m_c - \theta_c) + (1 - L)(1 - m_c)\right]. \end{cases} \tag{OA.4}$$

In the dirty-only regime, $m_c < 1/2$, $s_c = 0$, $s_d = 1 - L$, and

$$\begin{cases} \dot{\theta}_c &= -(\sigma - 1)\theta_c(1 - \theta_c)\mu(1 - L), \\ \dot{L} &= L\left[\mu L(1 + \alpha(1 - \theta_c)) - (\mu + \rho)\right] \\ \dot{m}_c &= -(1 - m_c)\mu\left[\alpha L(\theta_c - m_c) + (1 - L)m_c\right]. \end{cases} \tag{OA.5}$$

In the simultaneous regime, $m_c = 1/2$, $s_c = (1 - L)/2 + \alpha L(\theta_c - 1/2) = 1 - L - s_d$,

$$\begin{cases} \dot{\theta}_c &= (\sigma - 1)\theta_c(1 - \theta_c)\alpha\mu L(2\theta_c - 1), \\ \dot{L} &= L\left[\frac{1}{2}(1 + \alpha)\mu L - (\frac{1}{2}\mu + \rho)\right]. \\ \dot{m}_c &= 0. \end{cases} \tag{OA.6}$$

OA3 Planner's solution

From (7), we write the carbon stock S as the sum of a non-decaying stock S_1 and a decaying stock S_2, where $\dot{S}_1 = \phi_L E$ and $\dot{S}_2 = \phi_D E - \delta S_2$. To allow for symmetric expressions (across clean and dirty sectors), we write emissions as $E = a_c Y_c + a_d Y_d$ but we maintain our assumption $a_c = 0$. The current value Hamiltonian of the planner's problem is given by

$$\begin{aligned}\mathscr{H}^{sp} = & \ln\left[\exp(-\gamma(S_1+S_2-\bar{S}))C\right] + \Omega_C\left[\left(\sum_{j\in\{c,d\}} C_j^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - C\right] \\ & + \sum_{j\in\{c,d\}} \Omega_{Yj}\left[L_j^{1-\alpha}\int_0^1 q_{ji}x_{ji}^{\alpha}di - Y_j\right] + \sum_{j\in\{c,d\}} \zeta_j\left[Y_j - \int_0^1 q_{ji}x_{ij}di - C_j\right] \\ & + \sum_{j\in\{c,d\}}\int_0^1 \lambda_{ji}^s \mu s_{ji}\left(\int_0^1 q_{ji}di\right)di + \lambda_{S1}\phi_L \sum_{j\in\{c,d\}} a_jY_j + \lambda_{S2}\left[\phi_D\sum_{j\in\{c,d\}} a_jY_j - \delta S_2\right] \\ & + \zeta_L\left[1 - \sum_{j\in\{c,d\}} L_j - \sum_{j\in\{c,d\}}\int_0^1 s_{ji}di\right] + \sum_{j\in\{c,d\}}\int_0^1 \xi_{s_{ji}} s_{ji}di,\end{aligned} \tag{OA.7}$$

where C, C_j, Y_j, $\{x_{ji}\}_{i=0}^1$, L_j, $\{s_{ji}\}_{i=0}^1$ ($j \in \{c,d\}$) are the choice variables, $\{q_{ji}\}_{i=0}^1$ ($j \in \{c,d\}$) and S are the state variables, Ω_C and Ω_{Yj} are the shadow price associated with C and Y_j respectively, ζ_j and ζ_L are the shadow prices associated with the market clearing conditions, λ_{ji}^s, λ_{S1}, and λ_{S2} are the co-state variables, and finally, $\xi_{s_{ji}}$ are the shadow prices associated with the non-negativity constraints.

The FOCs are given by

$$\frac{\partial \mathscr{H}^{sp}}{\partial C}: \quad C^{-1} = \Omega_C, \tag{OA.8}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial C_j}: \quad \zeta_j = \Omega_C\left(C_j/C\right)^{-1/\sigma}, \tag{OA.9}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial x_{ji}}: \quad \zeta_j q_{ji} = \Omega_{Yj}\alpha L_j^{1-\alpha} q_{ji} x_{ji}^{\alpha-1}, \tag{OA.10}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial Y_j}: \quad \Omega_{Yj} = \zeta_j + a_j\left[\phi_L\lambda_{S1} + \phi_D\lambda_{S2}\right], \tag{OA.11}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial L_j}: \quad \zeta_L = \Omega_{Yj}(1-\alpha)Y_j/L_j, \tag{OA.12}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial s_{ji}}: \quad \zeta_L = \lambda_{ji}^s \mu Q_j + \xi_{s_{ji}}, \tag{OA.13}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial q_{ji}}: \quad \dot{\lambda}_{ji}^s = \rho\lambda_{ji}^s - \mu\int_0^1 \lambda_{ji}^s s_{ji}di - \Omega_{Yj}L_j^{1-\alpha}x_{ji}^{\alpha} + \zeta_j x_{ji}, \tag{OA.14}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial S_1}: \quad \dot{\lambda}_{S1} = \gamma + \rho\lambda_{S1}, \tag{OA.15}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial S_2}: \quad \dot{\lambda}_{S2} = \gamma + (\rho+\delta)\,\lambda_{S2}. \tag{OA.16}$$

From (OA.10) we conclude that $x_{ji} = x_j$ for all i. From (OA.13) we conclude that all producers i in sector j that are active in R&D have the same shadow price λ^s_{ji} denoted λ^s_j. Hence $\int_0^1 \lambda^s_{ji} s_{ji} di = \lambda^s_j s_j$, where s_j is aggregate R&D labor as above. Using this and $x_{ji} = x_j$ in (OA.14), we conclude that we can drop all i subscripts:

$$x_{ji} = x_j,\ s_{ji} = s_j,\ \lambda^s_{ji} = \lambda^s_j.$$

OA3.1 Social cost of carbon

Solving (OA.15) and (OA.16) we find $\lambda_{S1} = -\gamma/\rho$ and $\lambda_{S2} = -\gamma/(\rho+\delta)$, respectively. Hence, the shadow values of the two carbon stocks are constant (because of the logarithmic exponential structure as in Golosov et al. (2014)) and negative (because excess carbon causes climate damage and reduces welfare). We use Φ to denote the social cost of carbon emissions (in utility terms):

$$\Phi \equiv \phi_L(-\lambda_{S1}) + \phi_D(-\lambda_{S2}) = \gamma(\phi_L/\rho + \phi_D/(\rho+\delta)).$$

We define the social cost of emissions in sector j, in terms of j-goods, as

$$\tau^s_j \equiv a_j\,[\phi_L(-\lambda_{S1}) + \phi_D(-\lambda_{S2})]\,/\zeta_j.$$

Thus, we find that in the optimum the emission costs equal:

$$\tau^s_d = a_d\Phi/\zeta_d; \quad \tau^s_c = a_c\Phi/\zeta_c = 0, \tag{OA.17}$$

where we refer to τ^s_j as the tax and introduce the zero tax in the clean sector τ^s_c to allow symmetry in our expressions below.

OA3.2 Optimal input mix and static allocation

Because $x_{ji} = x_j$, the production function and goods market equilibrium can be written as, respectively, $Y_j = L_j^{1-\alpha} Q_j x_j^{\alpha} = C_j + Q_j x_j$. Substituting (OA.10) and (OA.11), we find expressions (36) for the production function and we find the consumption-output ratio C_j/Y_j as a function of the tax:

$$Y_j = Q_j L_j[\alpha(1-\tau^s_j)]^{\alpha/(1-\alpha)} \tag{OA.18}$$

$$C_j = Y_j[1-\alpha(1-\tau^s_j)]. \tag{OA.19}$$

Let $\chi_{C,j}$ denote the share of goods j in total value of consumption and χ_{Lj}

denote the share of production labor hired in sector j, that is

$$\chi_{C,j} \equiv \frac{\zeta_j C_j}{\Omega_C C}, \qquad \chi_{L,j} \equiv \frac{L_j}{L}. \tag{OA.20}$$

$\chi_{C,j}$ is thus the direct counterpart of the expenditure share in the decentralized equilibrium and $\chi_{L,j}$ the production labor share. From (OA.12), (OA.8), (OA.19), and (OA.20), we express the shadow price of labor as:

$$\zeta_L = \left(\frac{(1-\tau_j^s)(1-\alpha)}{1-\alpha(1-\tau_j^s)} \right) \frac{\chi_{Cj}}{L_j} \tag{OA.21}$$

We write the six equations (OA.12), (OA.10), (OA.9), (OA.11), (OA.18), and (OA.19) in relative terms (clean versus dirty) and solve for $\zeta_r, \Omega_{Yr}, C_r, Y_r, L_r, x_r$ in terms of τ_j^s and Q_r. Using these solutions with (20), the definition of θ_c, we find

$$\frac{\chi_{C,c}}{\chi_{C,d}} = \frac{\theta_c}{\theta_d}(1-\tau_d^s)^{-\frac{\sigma-1}{1-\alpha}}, \qquad \frac{\chi_{L,c}}{\chi_{L,d}} = \frac{\theta_c}{\theta_d}(1-\tau_d^s)^{-\frac{\sigma-\alpha}{1-\alpha}} \frac{1-(1-\tau_d^s)\alpha}{1-\alpha}. \tag{OA.22}$$

While in the decentralized equilibrium the relative technology fully captures the economic incentive for clean production and consumption, in the planner's solution these economic incentives must be augmented by the technology's contribution to carbon emission. Compared to the decentralized equilibrium, with the same level of relative technology θ_c, the planner will allocate more labor to the clean sector and consume a larger share of clean goods.

OA3.3 Static expression for the optimal tax

From (OA.8), (OA.9), (OA.20), and the definition of τ_d^s, we find $\tau_d^s = a_d \Phi / \zeta_d = a_d \Phi C_d / \chi_{C,d}$. Substituting $1/\chi_{C,d} = 1 + \chi_{C,c}/\chi_{C,d}$, (OA.19), (OA.18), and $L_d = \chi_{L,d} L$, we write:

$$\tau_d^s = a_d \Phi (\chi_{L,d} L) Q_d [\alpha(1-\tau_d^s)]^{\alpha/(1-\alpha)} (1 + \chi_{C,c}/\chi_{C,d})[1-\alpha(1-\tau_d^s)].$$

Since, from (OA.22) we find $\chi_{C,c}/\chi_{C,d} = (\chi_{L,c}/\chi_{L,d})(1-\tau_d^s)(1-\alpha)/(1-\alpha(1-\tau_d^s))$, we can write:

$$\tau_d^s = a_d \Phi L Q_d [\alpha(1-\tau_d^s)]^{\alpha/(1-\alpha)} \left[(1-\alpha(1-\tau_d^s))\chi_{L,d} + (1-\tau_d^s)(1-\alpha)\chi_{L,c} \right]. \tag{OA.23}$$

This equation gives a relationship between the tax and other key variables. We are interested in the relationship with θ_c, L, and Q_d. We therefore substitute $\chi_{L,d} =$

$1-\chi_{L,c}$ at the RHS and divide both sides by τ_d^s. It can be easily seen that then the RHS declines with τ_d^s and with $\chi_{L,c}$, where the latter itself increases with τ_d^s and θ_c. Hence, there is a unique solution for τ_d^s as a function of θ_c, L, and Q_d with the following properties:

$$\tau_d^s = \tilde{\tau}(\theta_c, a_d \Phi L Q_d) \in [0,1), \quad \tilde{\tau}_1 < 0, \quad \tilde{\tau}_2 > 0. \tag{OA.24}$$

OA3.4 Dynamic allocation

Time differentiating (20) and (OA.20), we find

$$\dot{\theta}_c = (\sigma-1)\theta_c(1-\theta_c)\left(\hat{Q}_c - \hat{Q}_d\right), \tag{OA.25}$$

$$\dot{\chi}_{C,c} = (\sigma-1)\chi_{C,c}(1-\chi_{C,c})\left[\hat{Q}_c - \hat{Q}_d + \frac{1}{1-\alpha}\frac{\tau_d^s}{1-\tau_d^s}\hat{\tau}_d^s\right], \tag{OA.26}$$

$$\dot{\chi}_{L,c} = (\sigma-1)\chi_{L,c}(1-\chi_{L,c})\left[\hat{Q}_c - \hat{Q}_d + \left(\frac{1}{1-\alpha} + \frac{1}{(\sigma-1)(1-\alpha(1-\tau_d^s))}\right)\frac{\tau_d^s}{1-\tau_d^s}\hat{\tau}_d^s\right]. \tag{OA.27}$$

From (OA.13) and the definition of m_j^s, (38), we derive as the planner's counterpart of (25) that sector j is the research-active sector if its social market value exceeds cost: $m_j^s > \kappa_j \Leftrightarrow s_j > 0$. Since $j = k$ denotes the research-active sector, we must have $m_k^s \geq \kappa_k$ and $\zeta_L = \mu\lambda_k^s Q_k$. This implies:

$$k = \begin{cases} c, & \text{if } m_c^s > 1/2; \\ d, & \text{if } m_c^s < 1/2. \end{cases} \tag{OA.28}$$

$$(\hat{Q}_c, \hat{Q}_d) = \begin{cases} (\mu(1-L), 0), & \text{if } k = c; \\ (0, \mu(1-L)), & \text{if } k = d. \end{cases} \tag{OA.29}$$

Substituting (OA.10), (OA.12), and (OA.18) into (OA.14) gives $\hat{\lambda}_j^s = \rho - \mu s_j - \zeta_L L_j / \lambda_j^s Q_j$. For the research active sector ($j = k$) we have $s_k > 0$ and $\zeta_L = \lambda_k^s \mu Q_k$ from (OA.13). Together with the definition of m_j^s, (38), this gives:

$$\hat{\lambda}_j^s = \rho - \mu s_j - \frac{m_k^s}{m_j^s}\mu L_j. \tag{OA.30}$$

For $j = k$, (OA.30) implies (37) in the main text. To derive the dynamics for m_j^s, note that $\hat{Q}_j = \mu s_j$ and $\hat{m}_k^s - \hat{m}_j^s = (\hat{\lambda}_k^s + \hat{Q}_k) - (\hat{\lambda}_j^s - \hat{Q}_j) = (m_k^s/m_j^s - L_k/L_j)\mu L_j$ where the second equality follows from (OA.30). Using $L_j = \chi_{L,j} L$ from (OA.20), we find

$$\dot{m}_c^s = \begin{cases} m_c^s(m_c^s - \chi_{L,c})\mu L, & \text{if } k = c; \\ 0, & \text{if } k = c,d; \\ (1-m_c^s)(m_c^s - \chi_{L,c})\mu L, & \text{if } k = d. \end{cases} \tag{OA.31}$$

To derive the dynamics for L, we combine (OA.13) and (37) to arrive at $\hat{\lambda}_k^s + \hat{Q}_k = \hat{\zeta}_L = \rho - \mu L_k$, while time differentiating (OA.21) implies $\hat{\zeta}_L = \hat{\chi}_{C,c} - \hat{\chi}_{L,c} - \hat{L}$. Hence, we arrive at $\hat{L} = \mu L_k - \rho + \hat{\chi}_{C,c} - \hat{\chi}_{L,c}$. Substituting (OA.26) and (OA.27), we find:

$$\dot{L} = L\left[\mu L_k - \rho + (\sigma-1)(\chi_{L,c} - \chi_{C,c})\left(\hat{Q}_c - \hat{Q}_d\right) \right. \\ \left. + \left(\frac{(\sigma-1)(\chi_{L,c}-\chi_{C,c})}{1-\alpha} - \frac{1-\chi_{L,c}}{1-\alpha(1-\tau_d^s)}\right)\frac{\tau_d^s}{1-\tau_d^s}\hat{\tau}_d^s\right]. \tag{OA.32}$$

Finally, we derive the dyamics of τ_d^s. From (OA.11) and (OA.12), we find $\zeta_d(1-\tau_d^s) = \zeta_L L_d/((1-\alpha)Y_d)$ and after using the definition of τ_d^s to eliminate ζ_d using (OA.13) and (OA.18) to eliminate ζ_L and Y_d, respectively, we derive

$$(1-\alpha)a_d\Phi = \tau_d^s[(1-\tau_d^s)\alpha^\alpha]^{-1/(1-\alpha)}\lambda_k^s\mu Q_k/Q_d.$$

Time differentiating this equation and substituting (OA.30) to eliminate $\hat{Q}_k$, we find the dynamics of the tax:

$$\dot{\tau}_d^s = -\frac{(1-\alpha)\tau_d^s(1-\tau_d^s)}{1-\alpha(1-\tau_d^s)}\left(\rho - \mu L_k - \hat{Q}_d\right). \tag{OA.33}$$

OA3.5 Characterising dynamics

Using (OA.22), (OA.24), and (OA.33) to eliminate χ_L, χ_C, τ_d^s, and $\dot{\tau}_d^s$, respectively, we find that (OA.25), (OA.29), (OA.31), and (OA.32) constitute four differential equations in four variables, namely θ_c, m_c, L, and Q_d.

The projection of the dynamics in the (θ_c, m_c^s) plane, used in Figure 8, can be characterised as follows. From (OA.28) we find the line $m_c^s = 1/2$ as the regime border which acts as the $\dot{\theta}_c = 0$ locus: above (below) the line there is clean (dirty) innovation only, and θ_c increases (decreases) over time if and only if $\sigma > 1$. Next, from (OA.31) we derive $m_c^s = \chi_{L,c}$ as the $\dot{m}_c^s = 0$ locus, with m_c increasing (decreasing) over time above (below) the locus. This locus is not a fixed line in the

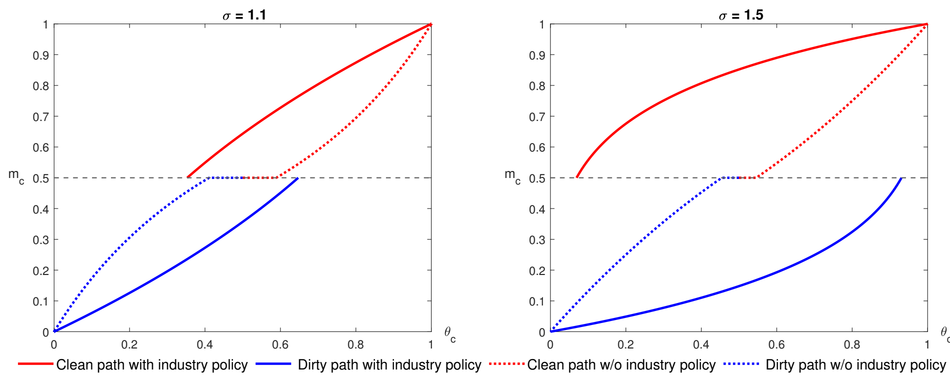

Figure 9: Size of overlap under industrial policy

plane, since $\chi_{L,c}$ depends on not only θ_c but also τ_d^s, see (OA.22), which depends on the whole dynamics of the system, cf. (OA.24). Nevertheless, (OA.22) shows that $\chi_{L,c} > \theta_c$ and $\chi_{L,c} \to \theta_c$ at the corners $\theta_c \to 0$ and $\theta_c \to 1$, so that the $\dot{m}_c^s = 0$ locus cuts the 45 degree line in the corners and is above the 45 degree line for θ_c. From (OA.25) and (OA.31), we derive that the slope of the optimum path, $\dot{m}_c^s/\dot{\theta}_c$, in the corner $(1,1)$ equals 0. Hence the optimal path must approach the clean steady state from the south west.

OA3.6 Regulated market economy

With a tax τ_E on carbon emission, the profit of final goods producers becomes

$$\pi_j = P_j Y_j - wL_j - \int_0^1 P_{ji} x_{ji} di - \tau_E a_j Y_j = (1-\tau_j) P_j Y_j - wL_j - \int_0^1 P_{ji} x_{ji} di, \tag{OA.34}$$

where $\tau_j = a_j \tau_E / P_j$ is the emission tax in terms of revenue (i.e. formulated as a value-added tax). Maximizing profits subject to the production function leads to a modified factor demand:

$$w = (1-\alpha)(1-\tau_j) P_j \frac{Y_j}{L_j}, \tag{OA.35}$$

$$P_{ji} = \alpha(1-\tau_j) P_j L_j^{1-\alpha} q_{ji} x_{ji}^{\alpha-1}. \tag{OA.36}$$

With this modified factor demand and the industry policy specified in the proposition, the Hamiltonian of the intermediate goods producers becomes

$$H_{ji} = (1+\tau_\alpha)\alpha(1-\tau_j)P_j L_j^{1-\alpha} q_{ji} x_{ji}^{\alpha} - P_j q_{ji} x_{ji} - w s_{ji} + \tau_{qj} q_{ji} + \lambda_{ji}\mu Q_j s_{ji}. \tag{OA.37}$$

where τ_α is the revenue subsidy, and τ_{qj} is the sector-specific technology subsidy. Accordingly, (10)-(12) change to

$$P_{ji} = \frac{1}{\alpha}(1+\tau_\alpha)^{-1} P_j q_{ji} = P_j q_{ji}, \tag{OA.38}$$

$$\mu Q_j \lambda_{ji} \leq w \perp s_{ji} \geq 0, \tag{OA.39}$$

$$\dot{\lambda_{ji}} = r\lambda_{ji} - \tau_{qj} - \frac{\partial \pi_{ji}}{\partial q_{ji}}. \tag{OA.40}$$

Combining the above results with (16) and (17), and using the symmetry result $x_{ji} = x_j, \lambda_{ji} = \lambda_j$, we can now summarize the regulated market economy by the following equations:

$$P_j/P = (C_j/C)^{-1/\sigma} \tag{OA.41}$$

$$1 = (1-\tau_j)\alpha L_j^{1-\alpha} x_j^{\alpha-1} \alpha(1+\tau_\alpha) \tag{OA.42}$$

$$w = (1-\tau_j)P_j(1-\alpha)Y_j/L_j \tag{OA.43}$$

$$w = \lambda_k \mu Q_k \tag{OA.44}$$

$$\hat{\lambda}_j - \hat{P} - \hat{C} = \rho - s_{qj}/\lambda_j - (1-\tau_j)P_j L_j^{1-\alpha} x_j^{\alpha}(1-\alpha)\alpha(1+\tau_\alpha) \tag{OA.45}$$

Combining the optimality conditions of the social planner's problem (OA.8)-(OA.14), using symmetry $x_{ji} = x_j, \lambda_{ji}^s = \lambda_j$ and the definition $\tau_j^s \equiv a_j\left[\phi_L(-\lambda_{S1}) + \phi_D(-\lambda_{S2})\right]/\zeta_j$, we find that the social planner's solution satisfies the following equations:

$$\zeta_j C = (C_j/C)^{-1/\sigma}, \tag{OA.46}$$

$$1 = (1-\tau_j)\alpha L_j^{1-\alpha} x_j^{\alpha-1}, \tag{OA.47}$$

$$\zeta_L = (1-\tau_j)\zeta_j(1-\alpha)Y_j/L_j, \tag{OA.48}$$

$$\zeta_L = \lambda_k^s \mu Q_k, \tag{OA.49}$$

$$\hat{\lambda}_j^s = \rho - \mu s_j - (1-\tau_j)\zeta_j L_j^{1-\alpha} x_j^{\alpha}(1-\alpha). \tag{OA.50}$$

Comparing (OA.46)-(OA.50) for the optimal economy to (OA.41)-(OA.45) for the regulated economy, we find that the latter replicates the former if the tax policies of proposition are imposed, $(1+\tau_\alpha)\alpha = 1$, $\tau_{qj} = ws_j/Q_j$, and $\tau_j = \tau_j^s$. Note that this implies $P_j/PC = \zeta_j$, $w/PC = \zeta_L$, $\lambda_j/PC = \lambda_j^s$, i.e. the real market prices in utility terms (market prices divided by P to make C the unit of account and then multiplied by marginal utility $1/C$ to make utility the unit of account) equal the corresponding shadow prices.

OA4 Cost of Policy Delay

We consider unanticipated policy delays. Figure 10 shows welfare of different transition paths with delayed policy as a percentage of the welfare maximum. The solid red line illustrates first-best policy after an initial delay. Compared to Acemoglu et al. (2012), who consider the cost of delay for second-best policy, the welfare cost of delay is relatively small under first-best policy. However, Figure 10 also shows that the welfare cost of policy delay increases with the length of delay. The increase in welfare cost is particularly prominent for the first-best policy. This suggests that, even if first-best policy is possible, policymakers should avoid a long delay.

Without a coordination device, the welfare cost of policy delay is best represented by the dotted blue line, which illustrates the most pessimistic scenario where the economy coordinates on delayed transition path with a lengthy transition delay. In this case, the welfare cost of delaying policy is substantial, ranging from more than 8% at a 10-year delay to 12% at a 50-year delay.

OA5 Segmented Labour Market

Suppose labour market is segmented and the total supply of labour is L for workers and s for scientists, that is, $L_c + L_d \leq L$ and $s_c + s_d \leq s$. sWage in (8) now differs from wage in (11). Denote the former by w_L and the latter by w_s. From (11) and (24), we see that we can continue to use the variable m_j to determine the innovation regime.

Combining (12), (15), (18) and (22), we find

$$\hat{\lambda}_j = r - (1-\alpha)\alpha \frac{P_j Y_j}{\lambda_j Q_j} = r - \frac{\alpha}{1+\alpha} \frac{\theta_j PC}{\lambda_j Q_j}. \tag{OA.51}$$

Set the final goods as numeraire so that $P = 1$ and define $I_j \equiv (\mu s Q_j \lambda_j)/C$. Note that I_j represents the investment rate in sector j, if innovation is in sector j only,

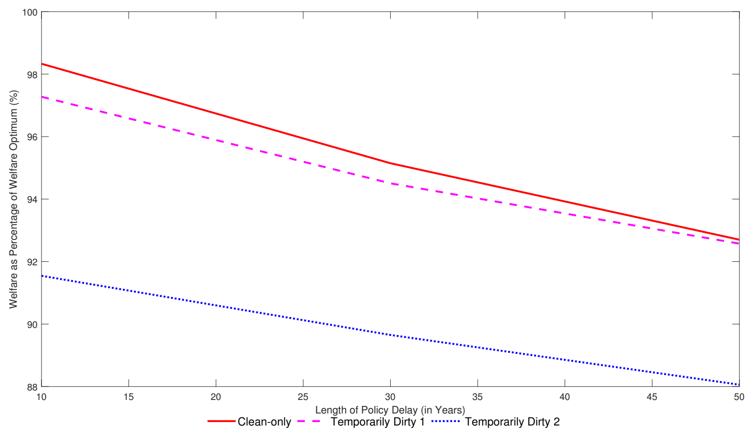

Figure 10: The Welfare Cost of Policy Delay

since in this case $I_j = \lambda_j \dot{Q}_j / C$. Combining the above equation with (14) and (17), we find

$$\hat{I}_j = \hat{\lambda}_j + \hat{Q}_j - \hat{C} = \rho - \frac{\alpha}{1+\alpha} \frac{\theta_j}{I_j} + \mu s_j. \tag{OA.52}$$

From (24), we find $\dot{m}_c = m_c(1-m_c)\left(\hat{I}_c - \hat{I}_d\right)$ or

$$\dot{m}_c = m_c(1-m_c)\left[-\frac{\alpha}{1+\alpha}\frac{\theta_c}{I_c}\left(1 - \frac{\theta_c}{1-\theta_c}\frac{m_c}{1-m_c}\right) + \mu s_c - \mu s_d\right] \tag{OA.53}$$

where s_j can be determined for each innovation regime.

Finally, the clean market share evolves according to $\dot{\theta}_c = (\sigma-1)\theta_c(1-\theta_c)(\mu s_c - \mu s_d)$. Together, $\dot{m}_c$, $\dot{m}_c$ and $\dot{I}_c$ form a differential equation system that summarizes the dynamics of the model. Note that assuming segmented labour market does not reduce the dimensionality of the model. The reason is that, as long as the investment decision is dynamic, the expected present value of investment m_j will be affected by the savings rate. Even though we have a fixed supply of scientists, the savings rate changes over time depending on the relative size of the two sectors and on the allocation of scientists. Thus, to reduce the dimensionality, we either need to make investment decision static (e.g. with one-period patent) or assume a fixed savings rate.

OA6 General Condition for Overlap

This appendix, first, generalizes the production and innovation technology to allow for more general complementarities in innovation and, second, shows that self-fulfilling prophecies (SFPs) can also arise when patents expire after one period.

OA6.1 Generalizing the sources of complementarity

We generalize the model in three ways to allow for multiple sources of investment complementarities. First, we allow a direct effect of intermediate firms' innovation on productivity in their sector, by generalizing the final good production to be

$$Y_j = (Q_j^{\varepsilon} L_j)^{1-\alpha} \int_0^1 q_{ji} x_{ji}^{\alpha} di, \tag{OA.54}$$

where $\varepsilon \geq 0$ measures how labor-augmenting the direct innovation spillovers are ($\varepsilon = 0$ brings us back to the main text model).

Second, we allow for a more general input-output structure by assuming intermediate goods production requires both own sector goods and general goods. The unit cost (or equivalently, its monopoly price divided by markup) of an intermediate in sector j with quality q_{ji} is now

$$q_{ji} P_j^{\omega} P^{1-\omega} = \alpha P_{ji}, \tag{OA.55}$$

where $\omega \geq 0$ measures the share of own sector inputs in the production of specific inputs ($\omega = 1$ brings us back to the main text model; Acemoglu et al (2012) choose $\omega = 0$).

Third, we allow intersectoral knowledge spillovers in innovation such that

$$\dot{q}_{ji} = \mu s_{ji} Q_j^{\eta+\chi} Q_{-j}^{\chi} (Q_c + Q_d)^{1-\eta-2\chi}, \tag{OA.56}$$

where $Q_c + Q_d$ is the general knowledge stock, χ is the degree of cross-sectoral spillovers and η denotes how much more own-sector knowledge enhances research productivity than other-sector knowledge; we have maintained the linear homogeneity that was also assumed in the main text. The model presented in the main text can then be considered a special case where $\eta = 1, \chi = 0$.

Lemma OA1. *In a static equilibrium, intermediate goods profits are linear in firms* *own quality* q_{ji}*, i.e.* $\pi_{ji} = \bar{\pi}_j q_{ji}$ *with*

$$\bar{\pi}_r = (Q_r)^{\psi}, \psi \equiv (1+\varepsilon)(\sigma-1)\frac{1-\alpha}{1-\omega\alpha} - 1, \tag{OA.57}$$

where $\bar{\pi}_r \equiv \bar{\pi}_c/\bar{\pi}_d$ *and* $Q_r \equiv Q_c/Q_d$*, while relative R&D costs are*

$$\frac{ws_{di}/\dot{q}_{di}}{ws_{ci}/\dot{q}_{ci}} = \mu_r(Q_r)^{\eta}. \tag{OA.58}$$

Proof of Lemma OA1. We determine static equilibrium, i.e. the allocation of labor and profits, given the state variables q_{ji} and given the amount of labor in production L.[1]

From demand for intermediates ($P_j \partial Y_j/\partial x_{ji} = P_{ji}$) and supply (OA.55) we find $x_{ji} = \alpha^{2/(1-\alpha)} Q_j^{\varepsilon} L_j (P_j/P)^{(1-\omega)/(1-\alpha)}$. Plugging this into the production function we find $Y_j = \alpha^{2\alpha/(1-\alpha)} Q_j^{\varepsilon+1} L_j (P_j/P)^{(1-\omega)\alpha/(1-\alpha)}$. Hence, in relative terms:

$$x_r = Q_r^{\varepsilon} L_r (P_r)^{(1-\omega)/(1-\alpha)},$$
$$Y_r = Q_r^{\varepsilon+1} L_r (P_r)^{(1-\omega)\alpha/(1-\alpha)}.$$

Demand for labor implies $P_j \partial Y_j/\partial L_j = P_j(1-\alpha)Y_j/L_j = w$, or in relative terms

$$L_r = P_r Y_r.$$

Demand for Y-goods implies:

$$Y_r = (P_r)^{-\sigma}.$$

Hence we have four equations in P_r, L_r, Y_r, x_r which can be solved in terms of Q_r.

$$P_r = (Q_r)^{-(\varepsilon+1)(1-\alpha)/(1-\omega\alpha)}$$
$$L_r = (Q_r)^{(\sigma-1)(\varepsilon+1)(1-\alpha)/(1-\omega\alpha)}$$
$$x_r = (Q_r)^{\varepsilon+(\varepsilon+1)[(\sigma-1)(1-\alpha)-(1-\omega)]/(1-\omega\alpha)}$$

Now we turn to profits of intermediate firms. Since the markup is $1/\alpha$, profits are $\pi_{ji} = (1-\alpha)P_{ji}x_{ji}$. and the price P_{ji} from (OA.55), we find $\pi_{ji} = [(1-\alpha)\alpha^{-1} x_{ji} P_j^{\omega} P^{1-\omega}] q_{ji} \equiv \bar{\pi}_j q_{ji}$, where the latter step uses the result that x_{ji} in equilibrium is the same across firms. This shows that profits are linear in own quality q_{ji}, which is stated in the lemma. Plugging in the solution for x_{ji} and taking relative

[1] Using this static allocation, below we turn to the dynamic equilibrium to determine the allocation of labor over production and innovation and the resulting dynamics of the state variable.

variables, we find $\bar{\pi}_r = x_r(P_r)^{\omega}$ which together with above solutions gives (OA.57).

From (OA.56) we directly find (OA.58). □

Hence ψ reflects investment complementarities in production: if $\psi > 0$, an increase in relative knowledge stocks increases relative marginal profits (in the main text, $\omega = 1, \varepsilon = 0$ so that $\psi = \sigma - 2$). Complementarities arise from (i) demand externalities (σ) (ii) input-output mulitipliers (ω) and (iii) direct productivity spillovers (ε). Furthermore, η reflect investment complementarities in innovation: if $\eta > 0$ investment in sector j reduces the cost of subsequent investment more than in the other sector.

Lemma OA2. *SFPs in the unregulated market economy require* $\psi > \max\{0, -\eta\}$.

Proof of Lemma OA2. This proof turns to the dynamics of the model and exploits the static equilibrium solutions in terms of the state variables q_{ji} from the revious proof. The intermediate good producer's investment problem of choosing s_{ji} has the following Hamiltonian:

$$H_{ji} = \bar{\pi}_j q_{ji} - w s_{ji} + \lambda_{ji} \mu \bar{Q}_j s_{ji}.$$

where $\bar{Q}_j = Q_j^{\eta+\chi} Q_{-j}^{\chi} (Q_j + Q_{-j})^{1-\eta-2\chi}$ is the productivity of research labor. The firm takes variables without i subscript as given. Optimality conditions are:

$$\bar{Q}_j \mu \lambda_{ji} \leq w \perp s_{ji} \geq 0 \tag{OA.59}$$

$$\hat{\lambda}_{ji} = r - \bar{\pi}_j / \lambda_{ji} \tag{OA.60}$$

The two conditions show that all firms within a sector have the same shadow value of quality, $\lambda_{ji} = \lambda_j$. We define two variables, z_c and m_c:

$$z_c \equiv \frac{\bar{\pi}_c \bar{Q}_c}{\bar{\pi}_c \bar{Q}_c + \bar{\pi}_d \bar{Q}_d} = \frac{(Q_r)^{\psi+\eta}}{1 + (Q_r)^{\psi+\eta}}, \tag{OA.61}$$

$$m_c \equiv \frac{\lambda_c \bar{Q}_c}{\lambda_c \bar{Q}_c + \lambda_d \bar{Q}_d} = \frac{\lambda_r (Q_r)^{\eta}}{1 + \lambda_r (Q_r)^{\eta}}. \tag{OA.62}$$

Variable z_c captures current (green) market conditions. It is a predetermined state variable, i.e. a transformation of the relative technology state variable Q_r. The transformation ensures that z_c captures all channels through which the state variable affects the return to innovation: complementarities in production (ψ) and in innovation (η). Variable m_c captures future (green) market conditions. It is a forward-looking variable constructed such that its value directly pins down which innovation is active. Clean (dirty) innovation requires future green market conditions to be sufficiently good (poor) according to:[2]

$$m_c > (<) 1/2 \Leftrightarrow \hat{Q}_r > (<) 0.$$

From optimality condition (OA.60) we derive the relative growth rates $\hat{\lambda}_r = \frac{\bar{\pi}_d}{\lambda_d}\left(1 - \frac{\bar{\pi}_r}{\lambda_r}\right)$ which in terms of our new variables reads:[3]

$$\hat{\lambda}_r = \left(\frac{\bar{\pi}_d/\lambda_d}{(1-z_c)m_c}\right)(m_c - z_c).$$

To derive the dynamics of the model in terms of z_c and m_c, we time differentiate (OA.61) and (OA.62):

$$\dot{z}_c = z_c(1-z_c)(\psi+\eta)\hat{Q}_r \tag{OA.63}$$

$$\dot{m}_c = m_c(1-m_c)(\hat{\lambda}_r + \eta\hat{Q}_r), \tag{OA.64}$$

We now build the phase diagram in (z_c, m_c) plane. The regime border is the horizontal line $m_c = 1/2$. We first consider $\psi + \eta < 0$ and show that this rules out SFPs. If $m_c < 1/2$, innovation is brown, Q_r declines and z_c grows. Symmetric for $m_c > 1/2$. Hence the interior steady state with simultaneous research and $m_c = z_c = 1/2$ is stable, the corner steady states can never be reached, and no SFPs can arise.

We next show that an overlap requires $\psi > 0$. Assume $\psi + \eta > 0$. The slope of any time path is given by $\dot{m}_c/\dot{z}_c$. On the 45 degree line (with $m_c = z_c$ and hence $\hat{\lambda}_r = 0$) this slope boils down to:

$$\left.\frac{\dot{m}_c}{\dot{z}_c}\right|_{m_c=z_c} = \frac{\eta}{\psi+\eta}. \tag{OA.65}$$

This means that, unless $\eta/(\psi+\eta) = 1 \iff \psi = 0$, an equilibrium path can cross the 45 degree line only once. When tracing back the equilibrium path from a corner steady state (either the dirty staedy state $m_c = z_c = 0$ or the clean one $m_c = z_c = 1$), we start on the 45 degree line and never cross again; when the slope is smaller than

[2]From (OA.59) we derive the regime border condition $\lambda_r \bar{Q}_r \mu_r > (<) 1 \Leftrightarrow \hat{Q}_r > (<) 0$ which in terms of m_c gives the expression.

[3]Note $\bar{\pi}_r/\lambda_r = z_r/m_r = [z_c/(1-z_c)]/[m_c/(1-m_c)]$.

1, the path from the dirty (clean) steady state crosses the regime border to the right (left) of the 45 degree line, implying an overlap. Hence the condition for SFPs is $\eta/(\psi+\eta)<1 \Leftrightarrow \psi>0, \psi+\eta>0$. □

Remark. This proof only uses the investment conditions and does not need consumer intertemporal utility maximization. This is becuase we only need to solve for relative variables. When we want to solve for all variables, in particular total - rather than relative - investment, as measured by $1-L$, we need the savings block of the model.

OA6.2 One-period Patents

We now modify the innovation part of the model. The production technology remains as in the main text, but we assume time is discrete and assume a patent length of one period only, as in Acemoglu et al. (2012). Suppose each period the intermediate good monopolists are each assigned a patent randomly that allows them to produce a particular good for one period. They can either produce at the given technology $q_{j,i,t-1}$ or improve their technology first by hiring scientists. The demand for their goods is still given by (9) and the quality of goods can be improved according to

$$q_{j,i,t} = q_{j,i,t-1} + \mu s_{j,i,t} Q_{j,t-1}. \tag{OA.66}$$

Since we maintain the production structure of the main model, (15) continues to hold at any period t. Using this expression and maximizing $\pi_{ji} - ws_{ji}$, we find that the investment decision is given by

$$\frac{\partial \pi_{j,i,t}}{\partial q_{j,i,t}} \mu Q_{j,t-1} \geq w_t \perp s_{j,i,t} \geq 0. \tag{OA.67}$$

Writing in relative terms and using (23) we find

$$\frac{\partial \pi_{c,i,t}/\partial q_{c,i,t}}{\partial \pi_{d,i,t}/\partial q_{d,i,t}} = \left(\frac{Q_{c,t}}{Q_{d,t}}\right)^{\sigma-2} > (<) \frac{\mu}{\mu} \left(\frac{Q_{c,t-1}}{Q_{d,t-1}}\right)^{-1} \Leftrightarrow s_{c,i,t} > (=)0, s_{d,i,t} = (>)0.$$

Substituting the aggregate innovation outcome

$$Q_{j,t} = Q_{j,t-1} + \mu s_{j,t} Q_{j,t-1} = (1+\mu s_{j,t}) Q_{j,t-1} \tag{OA.68}$$

and substituting (21) and (24), we find for regime switching

$$\frac{\theta_{c,t-1}}{\theta_{d,t-1}}\left(\frac{1+\mu s_{c,t}}{1+\mu s_{d,t}}\right)^{\sigma-2} \gtreqless 1. \tag{OA.69}$$

If $\sigma > 2$, the LHS of the above inequality is increasing in $s_{c,t}$ and decreasing in $s_{d,t}$. Self-fulfilling prophecies can thus arise. Let $s_t \equiv s_{c,t} + s_{d,t}$. If

$$(1+\mu s_t)^{-(\sigma-2)} \leq \frac{\theta_{c,t-1}}{\theta_{d,t-1}} \leq (1+\mu s_t)^{\sigma-2}, \tag{OA.70}$$

the selection of the innovation regime depends on agents' beliefs about $s_{c,t}$ and $s_{d,t}$. If $\theta_{c,t-1}/\theta_{d,t-1}$ is to the right of the right boundary of (OA.70), innovation can only occur in the clean sector; whereas if $\theta_{c,t-1}/\theta_{d,t-1}$ is to the left of the left boundary of (OA.70), innovation can only occur in the dirty sector. Thus, (OA.70) also provides the boundaries of the overlap. It is further clear that the size of the overlap tends to increase with σ.

OA6.3 Variable patent length

While Acemoglu et al. (2012) assume one-period patents and our main text model assumes infinite patent length, in reality patents often last between 15 and 20 years. To model elementary aspects of patent protection issues, we assume all intermediate firms face a risk of losing their profits permanently because of patent infringement.[4] The infringement event occurs at Poisson rate ι, so that the arbitrage equation (12) now contains a risk premium:

$$\dot{\lambda}_{ji} = (r+\iota)\lambda_{ji} - \frac{\partial \pi_{ji}}{\partial q_{ji}}, \tag{OA.71}$$

which implies that profits are discounted with the interest rate plus the infringement risk ι to calculate the value of investment λ_{ji}.

The patent infringement rate does not affect the analysis in Section 3. Intuitively, because patent infringement occurs with the same probability in all sectors, it does not affect the direction of investment.

However, the patent infringement rate affects the speed of overall investment as

[4] This modelling assumes that infringement is exogenous and uniform across firms; firms who "steal" the patent are immediately in the same position as robbed incumbent. A full modelling would require specifying who is successful in infringement, whether this costs effort etc. Moreover, (legal) patent length is not the same as (illegal) infringement. We leave these details for further research.

analysed in Section 4. This follows from combining (11), (OA.71), (15), (13), and (8), which gives

$$\hat{\lambda}_k = r + \iota - \alpha\mu L_k \quad \text{(OA.72)}$$

$$r = \alpha\mu L_k + \hat{w} - \hat{Q}_k - \iota. \quad \text{(OA.73)}$$

Following the same procedure as in the main text to derive the reduced-form equilibrium dynamics, we find that in (31) as well as in all equations in Lemma 1, ρ is replaced by $\rho + \iota$. Intuitively, a higher probability of loosing the patent right reduces investors' horizon as does an increase in the discount rate, so that the sum of discount rate and patent infringement rate governs the speed of innovation. The effect of a change in ρ and a change in ι are the same with respect to the equilibrium dynamics analysed in section 4. Hence, we conclude that a shorter average patent length (increase in ι) makes the overlap smaller.

In the market economy, policy is needed to counteract the excessively short horizon of investors introduced by effect of finite patent length. A subsidy to R&D can do this job and is needed to decentralize the first-best.